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Three-dimensional Analytical Models for Time-dependent Coefficients through Uniform and Varying Plane input Source in Semi-infinite Adsorbing Porous Media.

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ABSTRACT: In the present study, analytical solutions are developed for three-dimensional advection-dispersion equation (ADE) in semi-infinite adsorbing saturated homogeneous porous medium with time dependent dispersion coefficient. It means porosity of the medium is filled with single fluid(water). Dispersion coefficient is considered proportional to seepage velocity while adsorption coefficient inversely proportional to dispersion coefficient. Solutions are derived for both uniform and varying plane input source. The source geometry, including shape and orientation, broadly act major role for the concentration profile through the entire transport procedure. Initially the porous domain is not solute free. It means domain is throughout uniformly polluted. With help of certain transformation advection-dispersion equation is reduced into constant coefficient. The governing advection-dispersion equation, initial and boundary condition is solved by applying Laplace Transform Technique (LTT). The desired closed-form solution for the line source in two-dimensions and point source in one-dimension of uniform and varying nature are also evaluated as particular cases. Effects of parameters and value on the solute transport are demonstrated graphically.

Keywords: advection-dispersion equation; isotropic; saturated; homogeneous; plane source.

INTRODUCTION

In recent years, groundwater resources have become polluted due the human activities, production of chemically reactive contaminants from industrial and/or other waste, percolation of pesticides etc. Contaminants of groundwater penetrate through pores in the groundwater table and attenuate the pollution concentration as the pollutants cleave to the solid surface. Once groundwater polluted by any means it becomes very difficult to improve its quality. The transport of solutes in porous media is governed by advection-dispersion equation which is a parabolic type partial differential equation of second order. Analytical/numerical solutions along with an appropriate initial and boundary conditions help us to demonstrate the contaminant distribution in geological formations. Properties of geological formations, such as hydraulic conductivity and porosity, generally showing high degrees of spatial variation and this mutation majorly determine groundwater flow and solute transport phenomena. Analytical solutions of one, two and three-dimensional solute transport problems,

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subjected to various initial and boundary conditions, in finite, semi-finite porous domain have been published in the literature (e.g., Kumar & Kumar, 1998; Singh et al., 2008; Singh et al., 2009; Kumar et al., 2010; Yadav et al., 2011; Yadav et al., 2012; Chen & Liu, 2011). In order to deal solute pollution, it is necessary to understand the physical mechanism of mass transport in geological formation. Simunek & Suarez (1994) obtained two-dimensional solute transport for variable porous media. Yadav et al. (1990) obtained analytical solutions describing the concentration distribution along one-dimensional unsteady seepage flow through adsorbing saturated finite porous medium. Srinivasan & Clement, (2008) derived analytical solution for one-dimensional solute transport problem in porous domain. Sudicky & Cherry (1979) demonstrated that the contaminant dispersivity increased with distance from the contaminant source. Amro et al. (2012) observed that the effect of temporal variations on solute transport is disguised by the effect of spatial heterogeneity. Yim & Mohsen (1992) observed that the tidal effect on solute transport in homogeneous porous domain in one-dimensional flow. Wilson & Miller (1978) obtained the two-dimensional solute transport models for the continuous discharge of a point source. Leij et al. (1991) derived an analytical solution for three-dimensional flow and Singh et al. (2010) obtained analytical solution by using Laplace and Hankel transform techniques for two-dimensional advection-dispersion equation in cylindrical coordinates subjected to Dirichlet and mixed type inlet conditions. Chen et al. (2008) obtained analytical solution of advection-dispersion equation assuming the dispersion coefficient as spatially dependent asymptotic function. Kumar et al. (2010) obtained an analytical solution for the advection-diffusion equation subject to variable coefficients in semi-infinite media. Yates (1990) obtained an analytical solution for accounting the one dimensional solute transport in heterogeneous porous media with a spatially dependent dispersion. Eungyu Park & Hongbin Zhan (2001) obtained analytical solutions of contaminant transport using Green’s function method from one, two and three-dimensional aquifer. Batu & Genuchten (1990) developed a mathematical model subjected to Dirichlet and Cauchy type of boundary conditions in two-dimensional solute transport. Cirpka & Attinger (2003) demonstrated that dispersion is increased by the conjugation of spatial heterogeneity and temporal fluctuations. In subsurface, flow and transport phenomena are not only dependent on spatial heterogeneity but also dependent on temporal variability as well. Temporal variability occurs due to seasonal variation in water level (Wang & Tsay, 2001). Guerrero et al. (2009) provided analytical solutions for the advection-dispersion equation accounted to steady and transient flow field in finite domain. Smedt (2006) presented analytical solutions solute transport in rivers considering the effects of first order decay and transient storage. Sanskrityayn et al. (2016) developed analytical solution of advection-dispersion equation with space and time dependent dispersion using Green’s function. Majdalani et al. (2015) investigated analytical model of solute transport with scale dependent dispersion through heterogeneous porous media. Zhan et al. (2016) explored contaminant transport with one-, two-, and three-dimensional accounting for arbitrary shape sources. Singh & Chatterjee (2016) obtained three-dimensional solute transport phenomena considering arbitrary plane source. Van Genuchten et al. (2013) obtained one and multi-dimensional analytical solution for advection-dispersion equation accounting for zero order production.

In this study, an analytical solution for the three-dimensional ADE in Cartesian coordinates is derived in semi-infinite domain along the flow. The Laplace transform technique is applied to derive the exact analytical solutions. Initially the geological formation is supposed to be uniformly polluted. The input condition is considered uniformly continuous and of varying nature both. The effects of the parameters on the solute transport are studied separately with the help of graphs. The two coefficients (dispersion and seepage velocity) of the ADE is considered as functions time variable. Due to unsteadiness of the flow field the temporal dependence is considered. A general plane source is considered to demonstrate the impact of non-point sources in groundwater contamination problems. New space and time variables are introduced to reduce the variable coefficients into constant coefficients.

MATHEMATICAL FORMULATION AND ANALYTICAL SOLUTION

The problem is formulated mathematically as the plane source in three-dimensions. The domain is considered semi-infinite and the aquifer is initially not solute free. The geometry of the problem is shown in Fig.1. The advection-dispersion equation in three-dimensions may be written as follows:

$$R_x \frac{\partial c}{\partial t} + R_y \frac{\partial c}{\partial t} + R_z \frac{\partial c}{\partial t} = \left\{ \frac{\partial}{\partial x}\left(D_x \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial c}{\partial y} \right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial c}{\partial z} \right) \right\} - \left(u_x \frac{\partial c}{\partial x} - u_y \frac{\partial c}{\partial y} - u_z \frac{\partial c}{\partial z} \right) \quad (1)$$

where $c[\mathrm{ML}^{-3}]$ represents the solute concentration of the pollutant transporting along the flow field through the medium at a position $\left(x[L], y[L], z[L] \right)$ and time $\mathrm{t}[\mathrm{T}]$. $D_x[\mathrm{L}^2\mathrm{T}^{-1}]$, $D_y[\mathrm{L}^2\mathrm{T}^{-1}]$ and $D_z[\mathrm{L}^2\mathrm{T}^{-1}]$ are dispersion coefficients along the x, y, z axes respectively. $u_x[\mathrm{LT}^{-1}]$, $u_y[\mathrm{LT}^{-1}]$ and $u_y[\mathrm{LT}^{-1}]$ are the unsteady uniform seepage velocity along the x, y, z axes respectively. Retardation is also assumed unsteady and it’s components along x, y, z axes are R_x, R_y and R_z respectively. First term on the left hand side of the Eq. (1) is represents change in concentration with time in liquid phase. Eq. (1) describes the change of the concentration due to advective transport in directions of x, y and z axes. The effect of molecular diffusion is not taken into account due to dominance of the mechanical dispersion on the hydrodynamic dispersion during solute transport. The medium is supposed to have a uniform solute concentration c_i before an injection of pollutant in the domain. The input condition is considered of uniform and varying type. The right boundary is assumed that the rate of change of concentration is equal to zero at infinity in directions of x, y and z axes. Van Genuchten & Alves (1982) prompted that the Cauchy boundary is more realistic than the Dirichlet boundary. The uniform and varying type input are discussed in two separate cases.

The general plane source is introduced as the non-point source with the assumption that the geological formations are uniformly polluted and other end the concentration gradient is zero. The constant pollutant through plane is considered which reaches the water table due to infiltration.

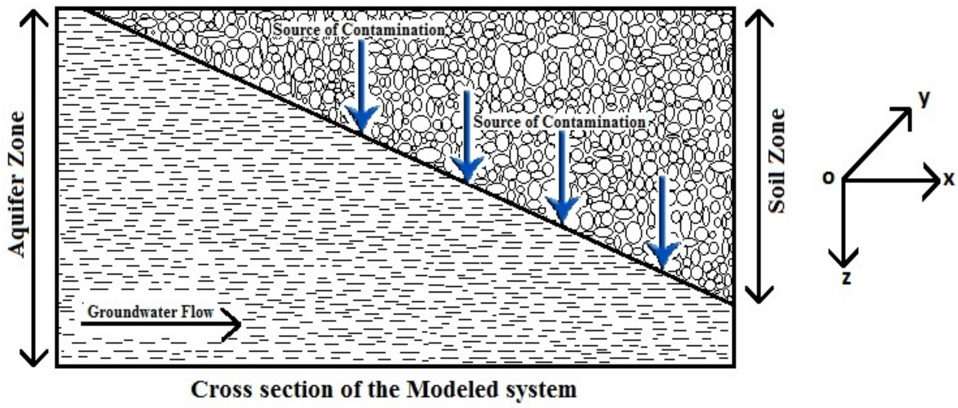

Fig. 1. Geometry of plane source contamination

The mathematical description of the cases follows as:

Case 1. Uniform input plane source condition:

Initial and boundary conditions follows as:

$$c(x,y,z,t)=c_i;t=0,\ x\geq 0,y\geq 0,\ z\geq 0 \tag{2}$$

$$c(x,y,z,t)=c_0;t>0\ ;\ a_1x+b_1y+e_1z=f \tag{3}$$

$$\frac{\partial c(x,y,z,t)}{\partial x}=0,\frac{\partial c(x,y,z,t)}{\partial y}=0,\frac{\partial c(x,y,z,t)}{\partial z}=0\ ;\ t\geq 0,x\to\infty,y\to\infty,z\to\infty \tag{4}$$

where c_i is solute concentration at origin. The constant contamination c_0 is considered through the general plane $a_1x+b_1y+e_1z=f$ as the boundary condition, where a_1, b_1, e_1 and f are arbitrary constants. The contaminant flux is zero when $x\to\infty, y\to\infty$ and $z\to\infty$. The dispersion, retardation and seepage velocity are temporally dependent along x,y,z axes respectively, are considered as follows:

$$u_x=u_{x0}\exp(-mt),\ u_y=u_{y0}\exp(-mt) \text{ and } u_z=u_{z0}\exp(-mt) \tag{5}$$

$$D_x=D_{x0}\exp(-mt),\ D_y=D_{y0}\exp(-mt) \text{ and } D_z=D_{z0}\exp(-mt) \tag{6}$$

$$R_x=\frac{R_{x0}}{\exp(-mt)},\ R_y=\frac{R_{y0}}{\exp(-mt)} \text{ and } R_z=\frac{R_{z0}}{\exp(-mt)} \tag{7}$$

where $m[T^{-1}]$ may be termed as an unsteady parameter. In order to remove the time function from coefficient of Eq. (1), a transformation is introduced as (Crank, 1975):

$$T=\int_0^t \exp(-2mt)dt \tag{8}$$

Eq. (1) and Eq. (2-4) reduce into following form:

$$R_{x0}\frac{\partial c}{\partial T}+R_{y0}\frac{\partial c}{\partial T}+R_{z0}\frac{\partial c}{\partial T}=D_{x0}\frac{\partial^2 c}{\partial x^2}+D_{y0}\frac{\partial^2 c}{\partial y^2}+D_{z0}\frac{\partial^2 c}{\partial z^2}-u_{x0}\frac{\partial c}{\partial x}-u_{y0}\frac{\partial c}{\partial y}-u_{z0}\frac{\partial c}{\partial z} \tag{9}$$

$$c(x,y,z,T)=c_i;T=0,\ x\geq 0,\ y\geq 0,\ z\geq 0 \tag{10}$$

$$c(x,y,z,T)=c_0;T>0,\quad a_1x+b_1y+e_1z=f \tag{11}$$

$$\frac{\partial c(x,y,z,T)}{\partial x}=0,\frac{\partial c(x,y,z,T)}{\partial y}=0,\quad \frac{\partial c(x,y,z,T)}{\partial z}=0;T\geq 0,x\to\infty,y\to\infty,z\to\infty \tag{12}$$

In order to reduce the Eq. (9) into single space variable, we take the following transformation (Singh et. al., 2016).

$$X=x+\frac{b_1}{a_1}y+\frac{e_1}{a_1}z \tag{13}$$

With transformation Eq. (13), Eq. (9) and Eqs. (10-12) change into following form:

$$R\frac{\partial c}{\partial T}=D_0\frac{\partial^2 c}{\partial X^2}-U_0\frac{\partial c}{\partial X} \tag{14}$$

where

$$\left.\begin{aligned} R&=R_{x0}+R_{y0}+R_{z0}\\ D_0&=D_{x0}+D_{y0}\frac{b_1^2}{a_1^2}+D_{z0}\frac{e_1^2}{a_1^2}\\ U_0&=u_{x0}+u_{y0}\frac{b_1}{a_1}+u_{z0}\frac{e_1}{a_1}\end{aligned}\right\} \tag{15}$$

$$c(X,T)=c_i;T=0,X\geq 0 \tag{16}$$

$$c(X,T)=c_0;T>0,X=\frac{f}{a_1} \tag{17}$$

$$\frac{\partial c(X,T)}{\partial X}=0;T\geq 0,X\to\infty \tag{18}$$

To eliminate the convective term from Eq. (14), we introduce following transformation:

$$c(X,T)=K(X,T)\exp\left\{\frac{U_0}{2D_0}X-\frac{U_0^2}{4RD_0}T\right\} \tag{19}$$

The Eq. (14) and Eqs. (16-18) reduce as:

$$R\frac{\partial K}{\partial T}=D_0\frac{\partial^2 K}{\partial X^2} \tag{20}$$

$$K(X,T)=c_i\exp\left\{-\frac{U_0}{2D_0}X\right\};T=0,X>0 \tag{21}$$

$$K(X,T)=c_0\exp\left\{-\frac{U_0}{2D_0}X+\frac{U_0^2}{4RD_0}T\right\};\quad T>0,X=\frac{f}{a_1} \tag{22}$$

$$\frac{\partial K}{\partial X}+\frac{U_0}{2D_0}K(X,T)=0;T\geq 0,X\to\infty \tag{23}$$

Taking the Laplace transformation, Eqs. (20-23) reduce into following boundary value problem.

$$\frac{d^2\overline{K}}{dX^2}-\frac{pR}{D_0}\overline{K}=-\frac{Rc_i}{D_0}\exp\left\{-\frac{U_0}{2D_0}X\right\} \tag{24}$$

$$\overline{K}(X,p)=\frac{c_0\exp\left\{-\frac{U_0}{2D_0}\frac{f}{a_1}\right\}}{\left(p-\frac{U_0^2}{4RD_0}\right)};X=\frac{f}{a_1} \tag{25}$$

$$\frac{d\overline{K}}{dX}+\frac{U_0}{2D_0}\overline{K}=0;X\to\infty \tag{26}$$

Solving Eqs. (24-26), we first obtain $\overline{K}(X,p)$ and then taking inverse Laplace transformation of $\overline{K}(X,p)$ and using Eq. (19) we get following solution:

$$c(X,T)=\left\{\frac{(c_0-c_i)}{2}\left[\begin{aligned}&\exp\left\{\frac{U_0^2}{4RD_0}T-\frac{U_0(X-a)}{2D_0}\right\}erfc\left\{\frac{(X-a)\sqrt{R}}{2\sqrt{D_0T}}-\frac{U_0\sqrt{T}}{2\sqrt{D_0}}\right\}+\\&\exp\left\{\frac{U_0^2}{4RD_0}T+\frac{U_0(X-a)}{2D_0}\right\}erfc\left\{\frac{(X-a)\sqrt{R}}{2\sqrt{D_0T}}+\frac{U_0\sqrt{T}}{2\sqrt{D_0}}\right\}\end{aligned}\right]\exp\left\{-\frac{U_0a}{2D_0}\right\}+c_i\exp\left\{-\frac{U_0X}{2D_0}+\frac{U_0^2}{4RD_0}T\right\}\right\}\exp\left\{\frac{U_0X}{2D_0}-\frac{U_0^2}{4RD_0}T\right\} \tag{27}$$

Particular sub cases:

Subcase1.1. Two dimensional uniform input line source solution:

Considering $D_{z0} = 0, u_{z0} = 0, R_{z0} = 0$ and $e_1 = 0$ in Eq. (27), we obtain the two-dimensional ADE solution for the line source of the contaminant. Solution may be written as:

$$c(x,y,T) = \left\{ \frac{(c_0 - c_i)}{2} \left[\begin{matrix} \exp\left\{ \frac{U^2}{4R'D}T - \frac{U\left(\left(x + \frac{b_1}{a_1}y\right) - a\right)}{2D} \right\} erfc\left\{ \frac{\left(\left(x + \frac{b_1}{a_1}y\right) - a\right)\sqrt{R'}}{2\sqrt{DT}} - \frac{U\sqrt{T}}{2\sqrt{D}} \right\} + \\ \exp\left\{ \frac{U^2}{4R'D}T + \frac{U\left(\left(x + \frac{b_1}{a_1}y\right) - a\right)}{2D} \right\} erfc\left\{ \frac{\left(\left(x + \frac{b_1}{a_1}y\right) - a\right)\sqrt{R'}}{2\sqrt{DT}} + \frac{U\sqrt{T}}{2\sqrt{D}} \right\} \end{matrix} \right] \exp\left\{ -\frac{Ua}{2D} \right\} + c_i \exp\left\{ -\frac{U\left(x + \frac{b_1}{a_1}y\right)}{2D} + \frac{U^2}{4R'D}T \right\} \right\} \tag{28}$$

$$\exp\left\{ \frac{U\left(x + \frac{b_1}{a_1}y\right)}{2D} - \frac{U^2}{4R'D}T \right\}$$

where, $D = D_{x0} + \frac{b_1^{\ 2}}{a_1^{\ 2}} D_{y0}$, $U = u_{x0} + \frac{b_1}{a_1} u_{y0}$ and $R' = R_{x0} + R_{y0}$

Subcase1.2. One-dimensional uniform input point source solution:

Now considering $D_{y0} = D_{z0} = 0, u_{y0} = u_{z0} = 0, R_{y0} = R_{z0} = 0, b_1 = e_1 = 0$ and $a = 0$ in Eq. (27), we obtain one-dimensional ADE solution for the point source of contamination as follows:

$$c(x,T) = \left\{ \frac{(c_0 - c_i)}{2} \left[\begin{matrix} \exp\left\{ \frac{u_{x0}^{\ 2}}{4R_{x0}D_{x0}}T - \frac{u_{x0}x}{2D_{x0}} \right\} erfc\left\{ \frac{x\sqrt{R_{x0}}}{2\sqrt{D_{x0}T}} - \frac{u_{x0}\sqrt{T}}{2\sqrt{D_{x0}}} \right\} + \\ \exp\left\{ \frac{u_{x0}^{\ 2}}{4R_{x0}D_{x0}}T + \frac{u_{x0}x}{2D_{x0}} \right\} erfc\left\{ \frac{x\sqrt{R_{x0}}}{2\sqrt{D_{x0}T}} + \frac{u_{x0}\sqrt{T}}{2\sqrt{D_{x0}}} \right\} \end{matrix} \right] + c_i \exp\left\{ -\frac{u_{x0}x}{2D_{x0}} + \frac{u_{x0}^{\ 2}}{4R_{x0}D_{x0}}T \right\} \right\} \tag{29}$$

$$\exp\left\{ \frac{u_{x0}x}{2D_{x0}} - \frac{u_{x0}^{\ 2}}{4R_{x0}D_{x0}}T \right\}$$

Case 2. Varying input plane source condition:

Uniform nature input condition on the earth surface exist at all time is not possible in real situations. In fact, due to human or other natural activities on the surface, the pollutant at the source may increase with time. Such situation may mathematically defined by mixed type condition. The present case may be defined by replacing the Eq. (3) by Eq. (30) in previous case

$$-D_x \frac{\partial c}{\partial x} - D_y \frac{\partial c}{\partial y} - D_z \frac{\partial c}{\partial z} + u_x c + u_y c + u_z c = (u_{x0} + u_{y0} + u_{z0})c_0 ; t > 0, a_1 x + b_1 y + e_1 z = f$$

Or

$$-D_{x0} \frac{\partial c}{\partial x} - D_{y0} \frac{\partial c}{\partial y} - D_{z0} \frac{\partial c}{\partial z} + u_{x0} c + u_{y0} c + u_{z0} c = \frac{(u_{x0} + u_{y0} + u_{z0})c_0}{\exp(-mt)} ; t > 0, a_1 x + b_1 y + e_1 z = f \tag{30}$$

Using a transformation Eq. (8), Eq. (30) reduces into following form:

$$-D_{x0}\frac{\partial c}{\partial x}-D_{y0}\frac{\partial c}{\partial y}-D_{z0}\frac{\partial c}{\partial z}+u_{x0}c+u_{y0}c+ \\ u_{z0}c=\left(u_{x0}+u_{y0}+u_{z0}\right)c_0\left(1-2mT\right)^{-\frac{1}{2}} \quad (31)$$

Transformation in Eq. (13) reduces Eq. (31) in to following form:

$$-D_{x0}\frac{\partial c}{\partial X}-D_{y0}\frac{\partial c}{\partial X}\frac{b_1}{a_1}-D_{z0}\frac{\partial c}{\partial X}\frac{e_1}{a_1}+ \\ u_0 c=u_0 c_0\left(1-2mT\right)^{-\frac{1}{2}} \quad (32)$$

Where $u_0 = u_{x0} + u_{y0} + u_{z0}$

Since for $m<<1$ the terms of order $o(m^2)$ or greater than it in binomial expansion of $(1-2mT)^{-\frac{1}{2}}$ are very small. Hence those terms can be eliminated from it. Eq. (32) reduces further into:

$$-D_1\frac{\partial c}{\partial X}+u_0 c=(1+mT)u_0 c_0 \quad (33)$$

Where, $D_1 = D_{x0} + D_{y0}\frac{b_1}{a_1} + D_{z0}\frac{e_1}{a_1}$

Using transformation Eq. (19) in Eq. (33), we get

$$-D_1\frac{\partial K}{\partial X}+\left(u_0-\frac{U_0 D_1}{2D_0}\right)K(X,T)= \\ (1+mT)u_0 c_0\exp\left\{-\frac{U_0}{2D_0}\frac{f}{a_1}+\frac{U_0{}^2}{4RD_0}T\right\}; \quad (34) \\ X=\frac{f}{a_1}, T>0$$

Taking the Laplace transformation of Eq. (34). it reduces into following form

$$-D_1\frac{\partial \bar{K}(X,p)}{\partial X}+\left(u_0-\frac{U_0 D_1}{2D_0}\right)\bar{K}(X,p)= \\ \frac{u_0 c_0\exp\left\{-\frac{U_0}{2D_0}\frac{f}{a_1}\right\}}{\left(p-\frac{U_0{}^2}{4RD_0}\right)}+\frac{mu_0 c_0\exp\left\{-\frac{U_0}{2D_0}\frac{f}{a_1}\right\}}{\left(p-\frac{U_0{}^2}{4RD_0}\right)^2}; \quad (35) \\ X=\frac{f}{a_1}$$

Now, we first solve boundary value problem represented by Eqs. (24, 26, 35) for value of $\overline{K}(X,p)$ and then solution of the problem is obtained using inverse Laplace transform of $\overline{K}(X,p)$ and Eq. (19). Solution of present problem is given by following equation.

$$c(X,T)=\left\{\begin{array}{l} (c_0-c_i)\frac{u_0}{D_1}\sqrt{\frac{D_0}{R}}\exp\left\{-\frac{U_0}{2D_0}a\right\}\left\{\frac{1}{2(\alpha+\beta)}E_1-\frac{1}{2(\alpha+\beta)}F+\frac{\beta}{\alpha^2-\beta^2}\exp\left\{\beta^2 T+\beta\sqrt{\frac{R}{D_0}}(X-a)\right\}erfc\left\{\frac{(X-a)\sqrt{R}}{2\sqrt{D_0 T}}+\beta\sqrt{T}\right\}\right\}+ \\ \frac{mu_0 c_0}{D_1}\sqrt{\frac{D_0}{R}}\exp\left\{-\frac{U_0}{2D_0}a\right\}\left\{\begin{array}{l}\frac{1-\left\{2\alpha T-\sqrt{R/D_0}(X-a)\right\}(\alpha+\beta)}{4\alpha(\alpha+\beta)^2}E_1+\frac{\left\{2\alpha T+\sqrt{R/D_0}(X-a)\right\}(\alpha-\beta)-1}{4\alpha(\alpha-\beta)^2}F-\frac{T}{\alpha^2-\beta^2}A+ \\ \frac{\beta}{\alpha^2-\beta^2}\exp\left\{\beta^2 T+\beta\sqrt{\frac{R}{D_0}}(X-a)\right\}erfc\left\{\frac{(X-a)\sqrt{R}}{2\sqrt{D_0 T}}+\beta\sqrt{T}\right\}\end{array}\right\}+ \\ c_i\exp\left\{-\frac{U_0 X}{2D_0}+\frac{U_0{}^2}{4RD_0}T\right\}\end{array}\right\} \\ \exp\left\{\frac{U_0 X}{2D_0}-\frac{U_0{}^2}{4RD_0}T\right\} \quad (36)$$

Where, $\alpha^2=\frac{U_0{}^2}{4RD_0}T$, $\beta=\frac{u_0}{D_1}\sqrt{\frac{D_0}{R}}-\frac{U_0}{2\sqrt{D_0 R}}$

$$A=\frac{1}{\sqrt{\pi T}}\exp\left\{\frac{-(X-a)^2 R}{4TD_0}\right\}$$

$$E_1=\exp\left\{\alpha^2 T-\alpha\sqrt{\frac{R}{D_0}}(X-a)\right\}erfc\left\{\frac{(X-a)\sqrt{R}}{2\sqrt{D_0 T}}-\alpha\sqrt{T}\right\}$$

$$F=\exp\left\{\alpha^2 T+\alpha\sqrt{\frac{R}{D_0}}(X-a)\right\}erfc\left\{\frac{(X-a)\sqrt{R}}{2\sqrt{D_0 T}}+\alpha\sqrt{T}\right\}$$

Particular Subcases:

Subcase 2.1. Two-dimensional varying input line source solution:

Considering $D_{z0} = 0, u_{z0} = 0, R_{z0} = 0$ and $e_1 = 0$ in Eq.(36),we obtain the two-dimensional ADE solution for the line source of the contaminations. Solution follows as:

$$c(x,y,T) = \left\{ \begin{array}{l} (c_0 - c_i)\dfrac{u_0'}{D_1'}\sqrt{\dfrac{D}{R'}}\exp\left\{-\dfrac{U}{2D}a\right\}\left\{\dfrac{1}{2(\alpha'+\beta')}E_1' - \dfrac{1}{2(\alpha'+\beta')}F' + \dfrac{\beta'}{\alpha'^2-\beta'^2}\exp\left\{\beta'^2 T + \beta'\sqrt{\dfrac{R'}{D}}\left(\left(x+\dfrac{b_1}{a_1}y\right)-a\right)\right\} erfc\left\{\dfrac{\left(\left(x+\dfrac{b_1}{a_1}y\right)-a\right)\sqrt{R'}}{2\sqrt{DT}} + \beta'\sqrt{T}\right\}\right\} + \\ \dfrac{mu_0'c_0}{D_1'}\sqrt{\dfrac{D}{R'}}\exp\left\{-\dfrac{U}{2D}a\right\}\left\{ \begin{array}{l} \dfrac{1-\left\{2\alpha'T - \sqrt{R'/D}\left(\left(x+\dfrac{b_1}{a_1}y\right)-a\right)\right\}(\alpha'+\beta')}{4\alpha'(\alpha'+\beta')^2}E_1' + \dfrac{\left\{2\alpha'T + \sqrt{R'/D}\left(\left(x+\dfrac{b_1}{a_1}y\right)-a\right)\right\}(\alpha'-\beta')-1}{4\alpha'(\alpha'-\beta')^2}F' - \dfrac{T}{\alpha'^2-\beta'^2}A' + \\ \dfrac{\beta'}{\alpha'^2-\beta'^2}\exp\left\{\beta'^2 T + \beta'\sqrt{\dfrac{R'}{D}}\left(\left(x+\dfrac{b_1}{a_1}y\right)-a\right)\right\} erfc\left\{\dfrac{\left(\left(x+\dfrac{b_1}{a_1}y\right)-a\right)\sqrt{R'}}{2\sqrt{DT}}\right\} \end{array} \right\} + \\ c_i \exp\left\{-\dfrac{U\left(x+\dfrac{b_1}{a_1}y\right)}{2D} + \dfrac{U^2}{4R'D}T\right\} \end{array} \right\} \tag{37}$$

$$\exp\left\{\dfrac{U\left(x+\dfrac{b_1}{a_1}y\right)}{2D} - \dfrac{U^2}{4R'D}T\right\}$$

Where, $A' = \dfrac{1}{\sqrt{\pi T}}\exp\left\{\dfrac{-\left(\left(x+\dfrac{b_1}{a_1}y\right)-a\right)^2 R'}{4TD}\right\}$

$$F' = \exp\left\{(\alpha')^2 T + \alpha'\sqrt{\frac{R'}{D}}\left(\left(x+\frac{b_1}{a_1}y\right)-a\right)\right\} erfc\left\{\frac{\left(\left(x+\frac{b_1}{a_1}y\right)-a\right)\sqrt{R'}}{2\sqrt{DT}} + \alpha'\sqrt{T}\right\}$$

$$E_1' = \exp\left\{(\alpha')^2 T - \alpha'\sqrt{\frac{R'}{D}}\left(\left(x+\frac{b_1}{a_1}y\right)-a\right)\right\} erfc\left\{\frac{\left(\left(x+\frac{b_1}{a_1}y\right)-a\right)\sqrt{R'}}{2\sqrt{DT}} - \alpha'\sqrt{T}\right\}$$

$\alpha'^2 = \dfrac{U^2}{4R'D}T$, $\beta' = \dfrac{u_0'}{D_1'}\sqrt{\dfrac{D}{R'}} - \dfrac{U}{2\sqrt{DR'}}$

$D_1' = D_{x0}\dfrac{\partial c}{\partial X} + D_{y0}\dfrac{b_1}{a_1}$, $D = D_{x0} + \dfrac{b_1^{\,2}}{a_1^{\,2}}D_{y0}$, $U = u_{x0} + \dfrac{b_1}{a_1}u_{y0}$

$R' = R_{x0} + R_{y0}$ and $u_0' = u_{x0} + u_{y0}$

Subcase 2.2. One-dimensional varying input point source solution:

Now considering $D_{y0} = D_{z0} = 0, u_{y0} = u_{z0} = 0, R_{y0} = R_{z0} = 0, b_1 = e_1 = 0$ and $a = 0$ in Eq. (36) we obtain one-dimensional ADE solution for the point source of contamination as follows:

$$c(x,T)=\left\{\begin{array}{l}(c_0-c_i)\dfrac{u_{x0}}{D_{x0}}\sqrt{\dfrac{D_{x0}}{R_{x0}}}\left\{TA''+\dfrac{1}{4\alpha''}E_1''-\dfrac{1}{4\alpha''}(1+2\alpha''\sqrt{R_{x0}/D_{x0}}\,x+4\alpha''^2 T)F''\right\}+\\ \dfrac{mu_{x0}c_0}{D_{x0}}\sqrt{\dfrac{D_{x0}}{R_{x0}}}\left\{\begin{array}{l}\dfrac{1}{16\alpha^3}\left(4\alpha''^2 T-2\alpha''\sqrt{R_{x0}/D_{x0}}\,x-1\right)E_1''\\ -\dfrac{1}{16\alpha''^3}\left(4\alpha''^2 T+2\alpha''^2\left(\sqrt{R_{x0}/D_{x0}}\,x+2\alpha''T\right)^2-1\right)F''+\dfrac{T}{4\alpha''^2}\left(1+\alpha''\sqrt{R_{x0}/D_{x0}}\,x+2\alpha''^2 T\right)A''\end{array}\right\}+\\ c_i\exp\left\{-\dfrac{u_{x0}x}{2D_{x0}}+\dfrac{u_{x0}{}^2}{4RD_{x0}}T\right\}\end{array}\right\} \tag{38}$$

$$\exp\left\{\frac{u_{x0}x}{2D_{x0}}-\frac{u_{x0}{}^2}{4R_{x0}D_{x0}}T\right\}$$

Where, $A''=\dfrac{1}{\sqrt{\pi T}}\exp\left\{\dfrac{-x^2R_{x0}}{4TD_{x0}}\right\}$

$$F''=\exp\left\{(\alpha'')^2T+\alpha''\sqrt{\frac{R}{D_{x0}}}x\right\}erfc\left\{\frac{x\sqrt{R}}{2\sqrt{D_{x0}T}}+\alpha''\sqrt{T}\right\}$$

$$E_1''=\exp\left\{(\alpha'')^2T-\alpha''\sqrt{\frac{R}{D_{x0}}}x\right\}erfc\left\{\frac{x\sqrt{R}}{2\sqrt{D_{x0}T}}-\alpha''\sqrt{T}\right\}$$

$$\alpha''^2=\frac{u_{x0}{}^2}{4R_{x0}D_{x0}}T$$

RESULTS AND DISCUSSIONS

The concentration values are evaluated in a finite region defined by $0\le x(km)\le 6$, $0\le y(km)\le 6$ and $0\le z(km)\le 6$. For numerical computation the plane sources is defined by taking $a_1=1, b_1=\sqrt{\dfrac{D_{y_0}}{D_{x_0}}}, e_1=\sqrt{\dfrac{D_{z_0}}{D_{x_0}}}$ for both uniform and varying plane input source condition. The two cases illustrated separately with the sub cases. For analysis of the results of the model with a plane source, numerical values of constants/parameters are taken from either published literature (Gelhar et al., 1992; Singh & Kumari, 2014) or experimental data. For example seepage velocity in porous media should be within the range $2m/day$ to $2km/year$ (Todd, 1980). The common parameters for both cases are taken as $c_0=1000mg/l$, $c_i=10mg/l$ and $m=0.01/year$. The Figures 2, 3, 4, 5 and 9, 10, 11, 12 are drawn with numerical values of parameters defined as: Initial seepage velocity along the x,y,z axes respectively are $u_{x0}=0.12km/year$, $u_{y0}=0.1km/year$, $u_{z0}=0.1km/year$ and dispersion profiles along the x,y,z axes respectively are $D_{x0}=0.0024km^2/year$, $D_{y0}=0.002km^2/year$, $D_{z0}=0.038km^2/year$ and $R_{x0}=0.68$, $R_{y0}=0.34$, $R_{z0}=0.23$ are the component of retardation along the x,y,z axes respectively and value of parameter $a=2.4$. Similarly the values of the parameters for the Figures 6, 7 and 13, 14 are defined as: $D_{x0}=0.024km^2/year$, $D_{y0}=0.002km^2/year$, $u_{x0}=0.12km/year$, $u_{y0}=0.1km/year$, $a=2.4$, $R_{x0}=0.78$ and $R_{y0}=0.47$. For point source Figure 8 and 15 dispersion $D_{x0}=0.11km^2/year$, seepage velocity $u_{x0}=0.01km/year$, $a=0$ and retardation $R_{x0}=1.15$.

Case 1. Three-dimensional line and surface graphs for uniform input plane source:

The demonstration of concentration profiles with distance along x, y and z axes respectively are shown in Figure 2. It is observed that the contaminant concentration decreases from a constant value and proceeds towards to zero. The decreasing rate of concentration with z axis is more rapid than the remaining two axes.

The solute concentration profile with the yz; zx and xy planes are also demonstrated and shown through surface graph in Figures 3-5 respectively. Figure 3 is drawn for fixed $z = 0.2$ that the contaminant concentration at $(0.2, 1.5382, 0.2)$ on the plane is 1 and starts decreasing along x and y axes directions. The contaminant concentration values decreases fast along x axis in comparison to y axis. It is also observed that on moving away from the plane the concentration reduces to its minimum level.

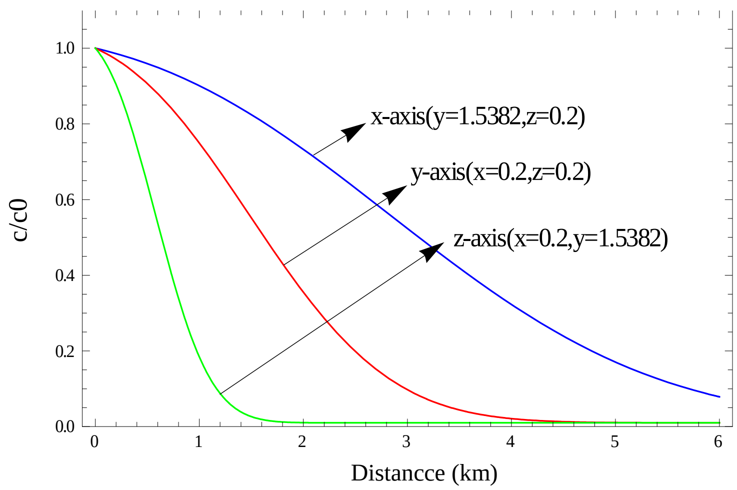

Fig. 2. Dimension less concentration profiles for uniform input plane source described by solution (27), at 5years

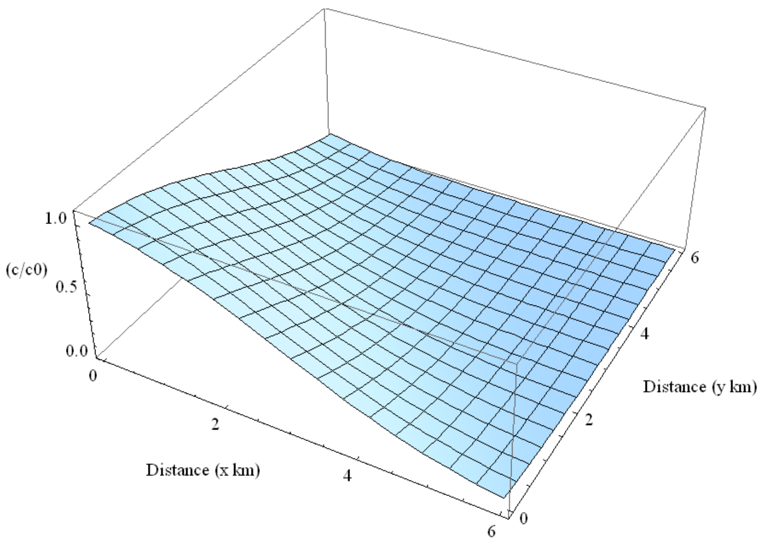

Fig. 3. Concentration distribution pattern in 2D space for uniform continuous input plane source described by solution (27), at 5 years

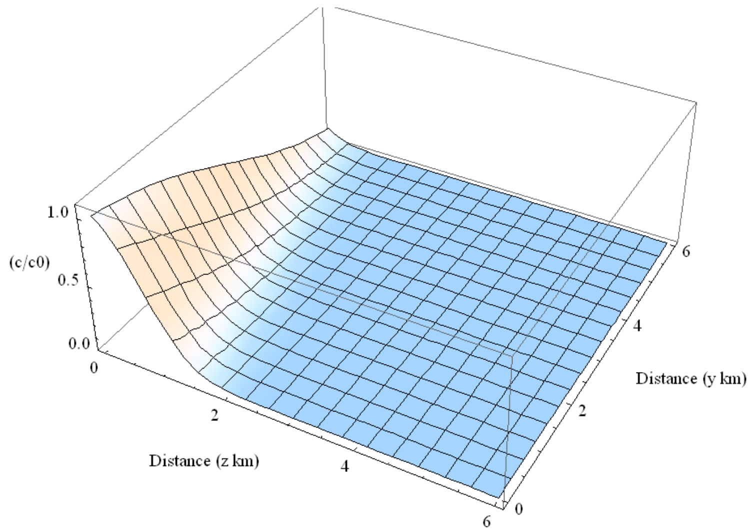

Fig. 4. Solute concentration in 2D space for plane source described by solution (27) at the end of 5 years

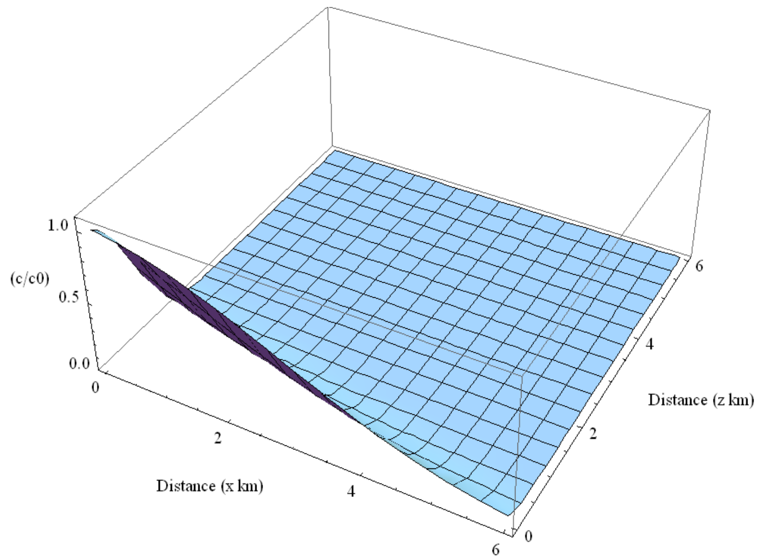

Fig. 5. Solute concentration in 2D space for plane source described by solution (27) at the end of 5 years

Figure 4 is drawn for fixed x coordinate at $x = 0.2$ in yz plane. On proceeding along z axis from point $(0.2,1.5382,0.2)$ on the plane decreasing rate of concentration is recorded high in comparison to moving along y axis. Like Figure 3 concentration decreases asymptotically on proceeding away from the plane.

Figure 5 is drawn for fixed y coordinate at $y = 1.5382$ in xz plane. On proceeding towards z axis from point $(0.2,1.5382,0.2)$ on the plane decreasing rate of concentration is recorded high in comparison to moving towards x axis.

The Figure 2 represents the line graph along the axes and Figures 3, 4, 5 are surface plots for the uniform plane source input. In Figure 2 it is observed that contaminant attenuation along z axis is faster in comparison to x and y axis but the trends are almost similar. Surface plot in Figure 3 explores for the fix depth (i.e. fixed z coordinate) attenuation increases slight as we move toward x axis. For fixed x distance attenuation is noticed fast closer to z axis. Again for the fixed y axis attenuation is recorded fast near z axis comparison to x axis. It may also be noticed that dimensionless concentration at plane is 1.

Subcase 1.1. Two dimensional line and surface graphs for uniform input line source:

The contaminant concentration profile with x and y are portrayed and shown in Figure 6. The pattern of contaminant concentrations are decreasing pattern with distance for obtained analytical solutions. Rehabilitations of contaminant concentration are similar along both the axis.

The change of contaminant concentration profiles are depicted for line-source is shown in Figure 7. The contaminant concentration value along both directions decreases with position. For example, if the septic tank or industrial garbage is situated away from certain distance of some contamination at certain depth then the source does not make any affect the quality of water.

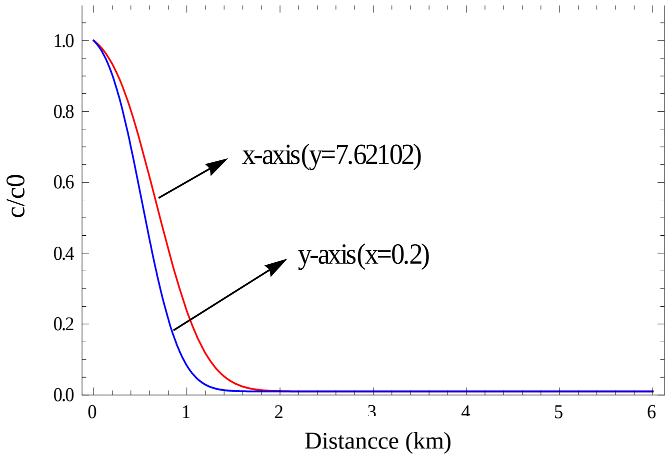

Fig. 6. Calculated contaminant concentration obtained with Eq. (28) for line sources along the distance x and y -axes at the end of 5 years

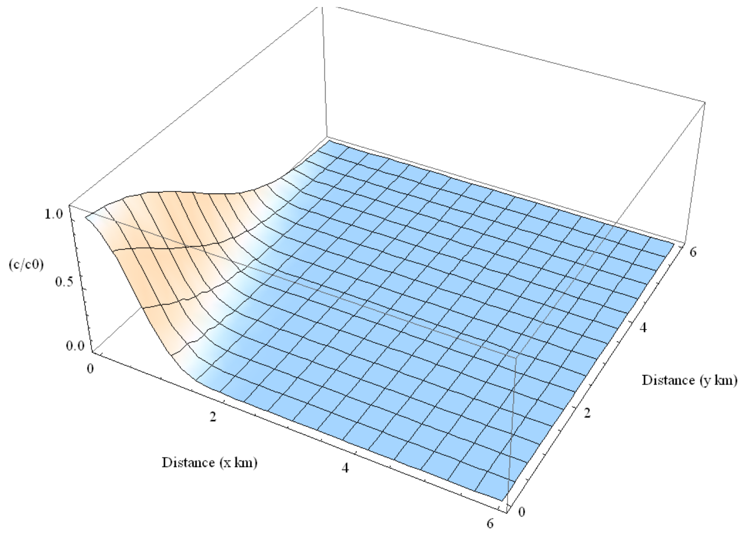

Fig. 7. Calculated contaminant concentration obtained with Eq. (28) for line sources at the end of 5 years

Subcase1.2. One dimensional point graph for uniform input point source:
Figure 8 illustrate the distribution of solute concentration at various times $t = 3,7$ and 11 years with same set of data. It is observed that the solute concentration rehabilitates up to same position for all times and attains minimum and harmless concentration, but near the source boundary the concentration values are little higher for higher time. The trend of contaminant concentration with time and distance travelled is almost identical at all times. The concentration values are changing with time and position.

Case 2. Three dimensional line and surface graphs for varying plane Input source:
Figure 9 illustrates the solute transport from the plane source along three directions described by the solution in Eq. (36). The input concentration, (c/c_0) at point $(0.2,1.5382,0.2)$ on plane is 0.9084 at time 5 years. It attenuates with position along x, y and z axes but it attenuates fastest along z axes.

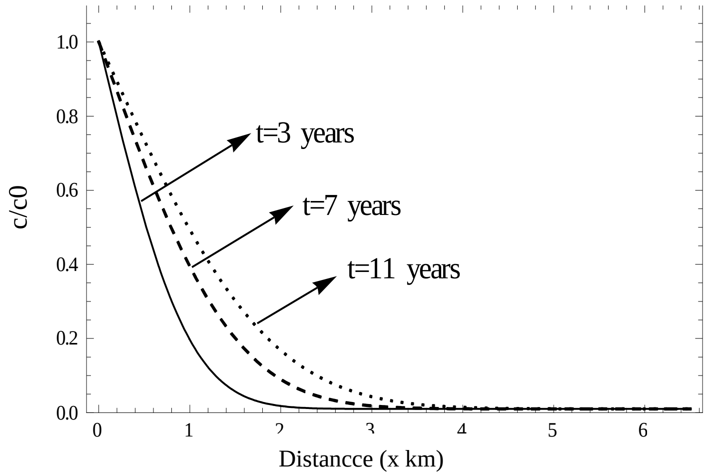

Fig. 8. Calculated contaminant concentration obtained with Eq. (29) for point sources at the end of $3,7,11$ years

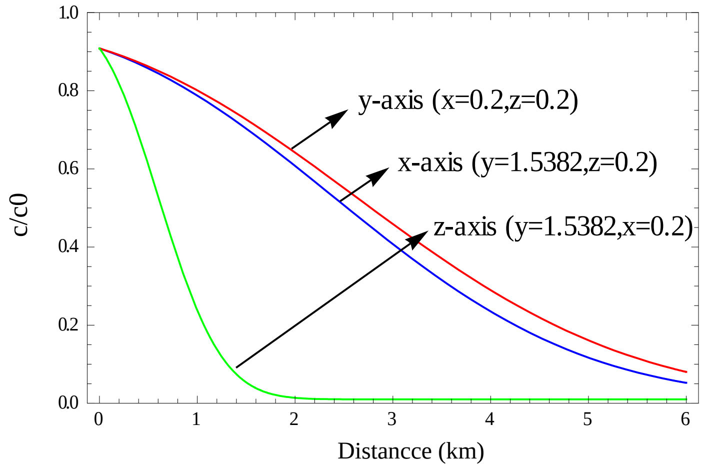

Fig. 9. Calculated contaminant concentration obtained with Eq. (36) for plane sources along the distance

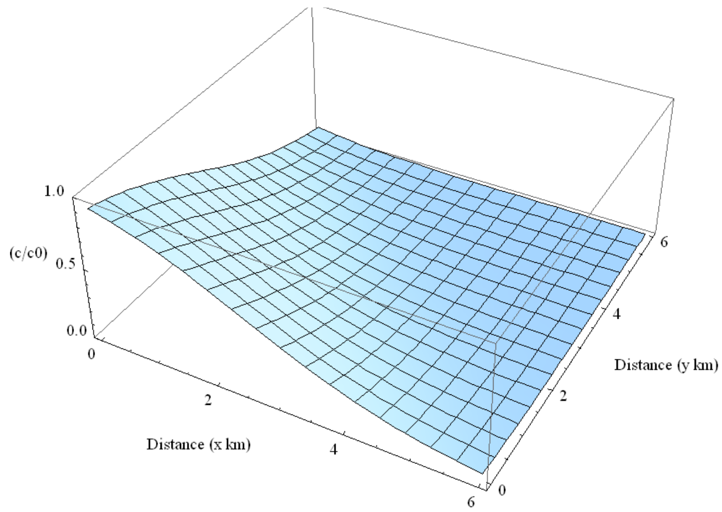

Fig. 10. Calculated contaminant concentration obtained with Eq. (36) for plane sources at the end of 5 years

Figure 10 demonstrates concentration profile in xy plane for fixed z coordinate (z= 0.2) at time *t= 5years*. Concentration at point $(0.2, 1.5382, 0.2)$ on the plane is recorded 0.9084 and on proceeding along both x and y axes directions it decreases. Rate of concentration decrement along y axis is fast in comparison to x axis. Concentration reduces to its minimum level on moving away from the plane.

Surface plot Figure 11 is drawn to illustrate the concentration pattern in zy plane for fixed $x = 0.2$.On moving away from the point $(0.2, 1.5382, 0.2)$ on the plane along z and y axis a rapid attenuation in concentration is observed along z axis than y axis.

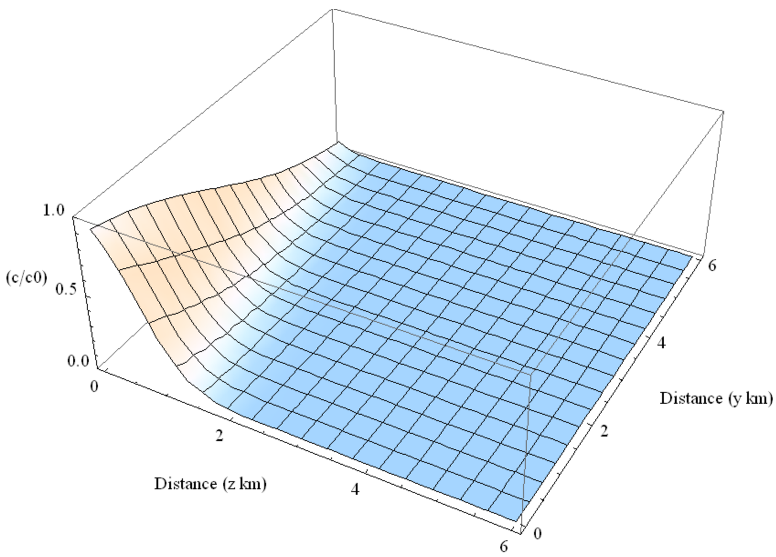

Fig. 11. Calculated contaminant concentration obtained with Eq. (36) for plane sources at the end of 5 years

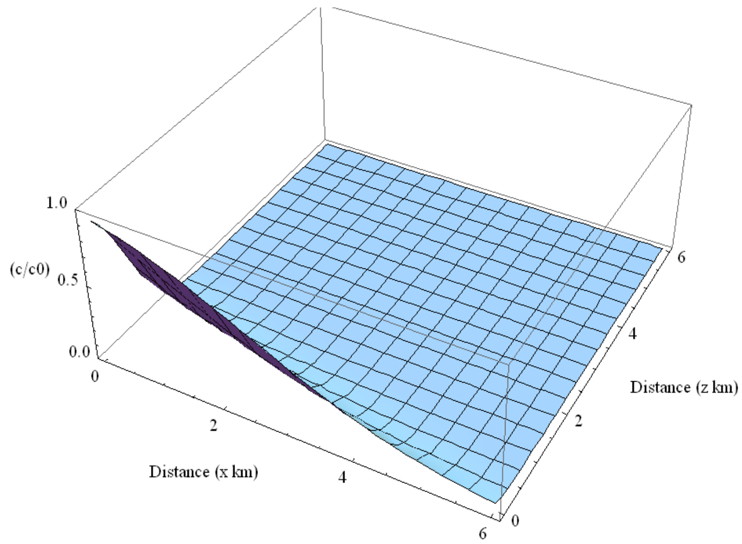

Fig. 12. Calculated contaminant concentration obtained with Eq. (36) for plane sources at the end of 5 years

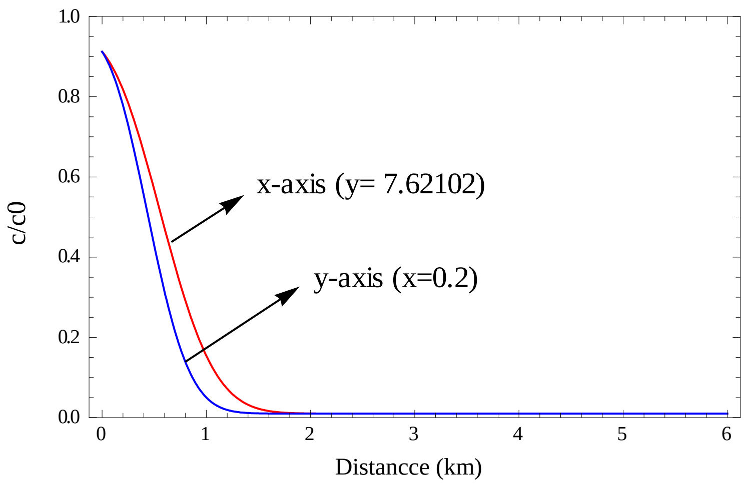

Fig.13. Calculated contaminant concentration obtained with Eq. (37) for line sources along the distance x and y -axes respectively at the end of 5 years

Figure 12 reveals the behavior of contaminant concentration in xz plane which is drawn for fixed y coordinate at $y = 1.5382$. Contaminant concentration c/c_0 on the point (0.2,1.5382,0.2) at left boundary lying on plane is evaluated 0.9084 and reduces continuously moving along x and z axis. The contaminant concentration values decreases fast along z axis in comparison to x axis. It is also observed that on moving away from the plane the concentration reduces to its minimum level.

Subcase 2.1. Two dimensional line and surface graphs for varying input line source:

The contaminant concentration profile with x and y are portrayed and shown in Figure 13. The pattern of contaminant concentrations are decreasing pattern with distance for obtained analytical solutions. Rehabilitations of contaminant concentration are similar along both the axis.

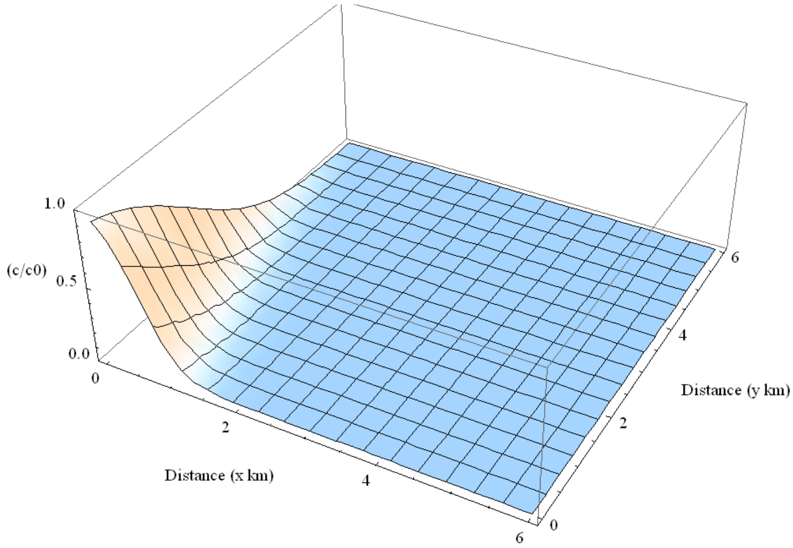

Fig. 14. Calculated contaminant concentration obtained with Eq. (37) for plane sources at the end of 5 years

The change of contaminant concentration profiles depicted for line-source is shown in Figure 14. The contaminant concentration value along both directions decreases with position. It may observe that the contaminant near x axis attenuates faster in comparison to y axis.

Subcase 2.2. One dimensional point graph for varying input point source: Figure 15 illustrates the solute transport from the point source of increasing nature along the longitudinal direction of the medium, described by the solution in Eq. (38). The input concentrations, c/c_0 at the origin are different at each time. It attenuates with position and time but near the boundary, concentration level is lower for lower time and higher for higher time.

Contaminant concentration attenuates and rehabilitate up to distance 2.5 km from source boundary.

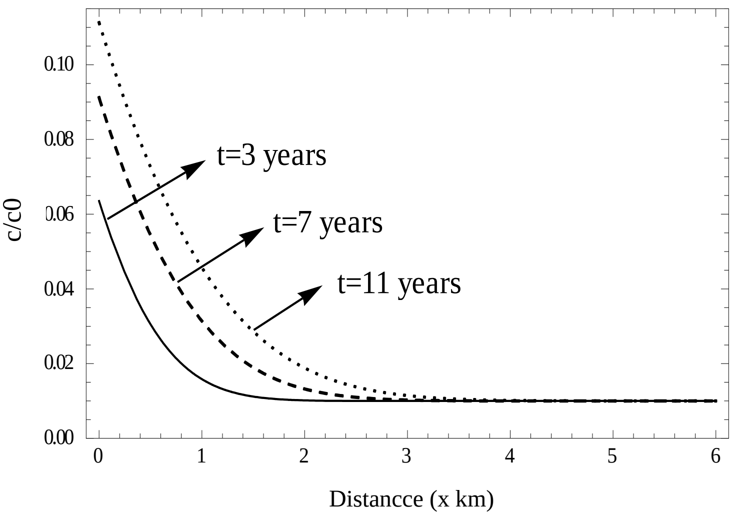

Fig. 15. Calculated contaminant concentration obtained with Eq. (38) for point sources at the end of 3, 7, 11 years

CONCLUSION

Analytical solutions to three dimensional advection-dispersion equation with variable coefficients along with initial and boundary conditions in a semi-infinite domain have been obtained. Laplace transformation technique has been used in getting the analytical solutions. With the help of new transformation the variable coefficients of the advection-dispersion equation are converted into constant coefficients. Effect of non-point source concentration for example plane and line source are discussed. In both cases (uniform and varying input source) the later components of velocity are taken into account. The effects of adsorption and other parameters on the solute transport are shows with graphs. Dispersion coefficient and the flow velocity are considered temporally dependent. The flow velocity is considered directly proportional to diffusion parameter and adsorption parameter is inversely proportional to diffusion parameter.

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