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RESEARCH PAPER

Study of Pollutant Dispersion in Finite Layers of Semi-infinite Geological Formation

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ABSTRACT
The present study deals with groundwater pollution in multilayer aquifer. The model is based on decomposition of finite layers in semi-infinite groundwater reservoir. A constant pollutant source is injected at the input boundary of the uppermost layer (UML) of the landfill. At the intermediate inlet boundary, some average value for the longitudinal exchange of the input source concentration in each sub-layer is considered from the previous layer. Initially, the aquifer is not solute free in each sub layer that means some constant background contaminant concentration exists. In each sub layer, concentration gradient is assumed to be zero at the extreme boundary. The linear sorption and first orders decay terms are considered to model the groundwater pollution in multilayer aquifer. The Laplace transform technique is adopted to solve one-dimensional (1D) advection-dispersion equation (ADE). This approach is helpful to understand the solute migration in finite sub layers. The results are elucidated for the different time periods to examine the peak of pollutant concentration level in geological formations.

KEYWORDS: Advection; Dispersion; Pollutant transport; Multilayer aquifer; porous media.

INTRODUCTION

Study of pollutant transport proposed since several decades through various mathematical, numerical and experimental approaches to predict the concentration distribution pattern in subsurface regions (Ebach & White, 1958; Corey et al., 1970). The analytical solutions were discussed for the various types of advection-dispersion equations (Gershon & Nir, 1969). In their study, they explored the effects of initial and boundary conditions for the distribution of tracer in time and space domain for several one-dimensional systems such as (infinite, semi-infinite, and finite). A mathematical study for the purely advective multilayer finite porous media were developed by using Laplace transforms technique (Higashi & Pigford, 1980). Later, other mathematical solutions were explored for the advective–dispersive equation in the multilayer's aquifer (van Genuchten, 1985). He developed few results for the movement of the contaminant in one-dimensional semi-infinite field with the help of Laplace transform and obtained the solutions up to four species using either first-type (Dirichlet) or third-type (Cauchy) inlet boundary conditions. Similarly, the analytical and numerical solutions were demonstrated for the solute transport in saturated porous media with the semi-infinite or finite thickness (Gelher & Collins, 1971; Sim & Chrysikopoulos, 1999; Smedt, 2006; Srinivasan &

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Clement, 2008). The groundwater flow and radionuclide movement in single fractures with diffusion was discussed in (Saied & Khalifa, 2002). The solute transport along with the longitudinal dispersion and time-dependent source condition was discussed in a semi-infinite aquifer with the unsteady groundwater flow, where dispersion and velocity was considered as functions of time variable (Singh et al., 2014). The pollutant's horizontal dispersion was demonstrated along and against sinusoidally varying velocity from a pulse type point source (Singh et al., 2015). Further, a study regarding the dispersion of depth dependent source in the two dimensional (2D) homogeneous porous medium was established in (Chatterjee & Singh, 2018). The arbitrary inlet boundary conditions were used to obtain a generalised analytical solution for the pollutant transport (Chen & Liu, 2012; Chen et al., 2012). Recently, the pollutant dispersion in semi-infinite porous medium were analysed with the impact of source sink term (Kumar et al., 2020) and the analytical solution regarding the pollutant transport along and against the groundwater flow was discussed in a semi-infinite porous medium (Singh et al., 2020).

The pollutant migration in multilayer aquifer via experimental, analytical and numerical simulations is rarely available in the hydrological literature. The main concern of this study is to examine how groundwater is contaminated through the soil, how pollutant would reach to the different zones of the compacted layer aquifer and how it may affect the groundwater available in pore spaces of the geological formations. As the migration of the pollutant through relatively impermeable soils is quite slow and therefore, the time required to reach the groundwater reservoir may ranges from several to hundreds of years. The 1D pollutant transport was discussed in the soil of finite depth (Rowe et al., 1985) and they discussed the pollutant transportation through clay layer of the finite depth. For the most practical situation, it was observed that the pollutant within the groundwater beneath of the landfill reached to a peak value at a specific time and then decreased with subsequent time. An analytical solution was studied for the one-dimensional solute transports with constant concentration at inlet boundary in the finite layer media (Alniami & Rustan, 1979) and adopted Laplace transform technique to develop the analytical solution without sorption and decay. 1D analytical and numerical results were proposed in two-layer porous media (Leij & Van Genuchten, 1995) and observed that the use of the Laplace transform technique become more complicated to predict the contaminant pattern at the interface boundary. An integral transform technique was used to predict the analytical solutions for the pollutant transport in porous media (Liu et al., 1998). The solute transport was investigated in the two-layer porous medium which was separated diagonally (Ghamariadyan et al., 2016) and discussed the effect of various parameters on the transport processes.

Advection-diffusion equation is not only applicable to study solute transport behaviour in groundwater but also in many other fields such as heat transport, air pollution modelling and many others fields. Air pollution modelling was discussed with the planetary boundary layer via discretization in sub layer (Vilhena et al., 1998). In each sub layer, the advection- diffusion equation was solved by Laplace transform technique. The one-dimensional solute transport was studied in double layer finite porous medium using finite element approach (Li & Cleall, 2011). The solution was described with five different scenarios with various combinations of fixed concentration, fixed flux and zeros concentration gradient at the input and output boundaries. ADE with the first-order decay was solved analytically for multilayer media using the classical integral transform technique (Guerrero et al., 2013). A case study was presented for the perimeter drilling survey regarding lakeside municipal solid wastage landfill (Yoshida et al., 2002) and obtained three consecutive aquifers isolated with each other at certain depth in that landfill. Pollutant was found extremely high in the uppermost sub layer compare to others.

In all the multilayer dispersion model as discussed earlier, mostly constant flow and constant dispersion were assumed. The present study is proposed to study the pollutant migration behaviour through the agricultural, industrial, domestic and other sources to the groundwater reservoir and subsequently in multilayer aquifer. The analytical solution is derived for the 1D ADE under sorption and first-order decay in a homogeneous multilayer porous media with time-dependent dispersion and flow velocity. Initially, aquifer is not assumed to be pollutant free and therefore, some constant background concentration may exist in each sub layer. Point source is considered at the inlet of the upstream boundary of the UML and some average value of concentration is considered from the previous layer at the intermediate inlet boundary for the other layers of the aquifer. Pollutant source is highly affected the upper layer of aquifer compare to other intermediate layer only because the distribution of contaminant decreases with the space variable. A semi-infinite aquifer is discretized into sub layers accordingly.

SYMBOLIC DISCRIPTION

$c_N(x, T)$	Plume concentration in suspension (liquid phase) $[ML^{-3}]$ for the N^{th} layer
x, t	Space $[L]$ and time $[T]$ variable
R_N	Retardation factor for the N^{th} level (dimensionless quantity)
$D_N(t)$	Longitudinal dispersion coefficient $[L^2T^{-1}]$ for the N^{th} layer
D_{0N}	Initial dispersion coefficient $[L^2T^{-1}]$ for the N^{th} layer
C_0	Input source concentration $[ML^{-3}]$ for the UML
c_{Ni}	Initial background contaminant concentration $[ML^{-3}]$ for the N^{th} layer
$U_N(t)$	Groundwater velocity $[LT^{-1}]$ for the N^{th} layer
U_{0N}	Initial groundwater seepage velocity $[LT^{-1}]$ for the N^{th} layer
$\Psi(\upsilon t)$	Groundwater velocity pattern, $\upsilon[T^{-1}]$
λ_N	The first-order decay rate $[T^{-1}]$ for the N^{th} layer
K_d	Solute solid-phase concentration $[MM^{-1}]$under sorption condition
ϕ_N	The porosity of the N^{th} layer
ρ	The density $[ML^{-3}]$of geological landfill

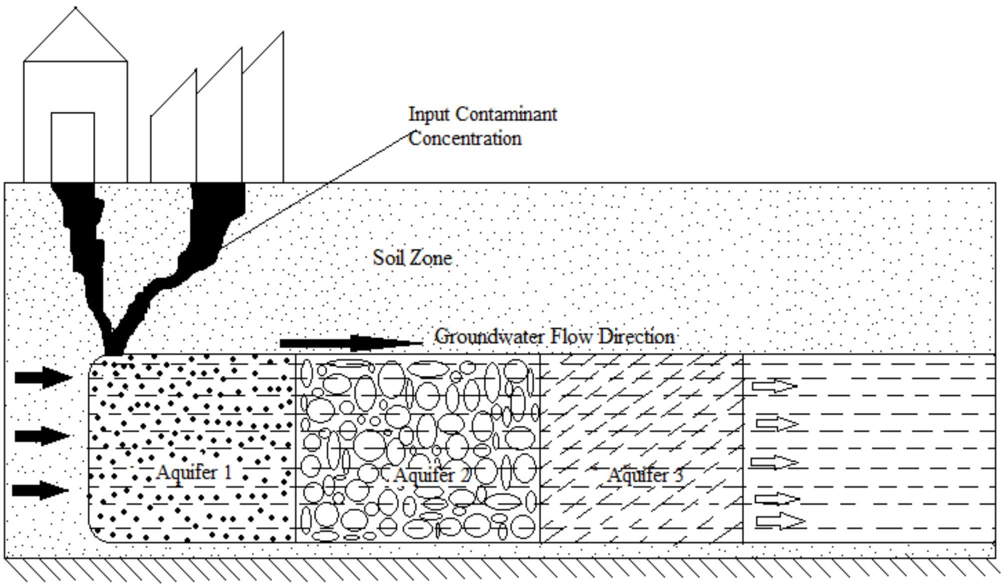

Figure 1. Geometry of the proposed model for the 1D multilayer's semi-infinite porous medium.

MATHEMATICAL FORMULATION

The geometry of the model problem is shown in Figure 1. The proposed model for the multilayer aquifer is developed with the help of Darcy's law. Each finite homogeneous layer with unsteady pore-water flow velocity $U_N(t)$ is considered. The positive x coordinate is measured along the pore-water flow direction, and the initial boundary is chosen at the origin of the coordinate system. Each layer has its own temporally dispersion coefficient $D_N(t)$ with retardation factor R_N and porosity ϕ_N. The subscript N represents the number of layers. Unsteady 1D advection-dispersion transport equation for the quantity $c_N \equiv c_N(x,t)$ in a semi-infinite porous media with some hydrological properties in each level is described as:

$$R_N \frac{\partial c_N}{\partial t} = D_N(t)\frac{\partial^2 c_N}{\partial x^2} - U_N(t)\frac{\partial c_N}{\partial x} - \lambda'_N c_N \tag{1}$$

Subscript $N = 1, 2, 3, 4.....n$ is denoted the sub layer of the geological landfill.

In each sub-layer, an average dispersion D_N and seepage velocity U_N are assumed (Ogata, 1970). Here dispersion varies as the first power of seepage flow velocity. Hence, the dispersion and seepage velocity are expressed as follows:

$$U_N = U_{0N}\psi(\upsilon t) \text{ and } D_N = D_{0N}\psi(\upsilon t) \tag{2}$$

$\psi(\upsilon t)$ is assumed as an exponentially decaying sinusoidal function of time t, with the flow resistance coefficient υ. Retardation factor of each sub layer of the groundwater under the linear sorption process may occur when pollutant enters into the soil water reservoirs.

Here, $R_N = 1 + \rho_N K_d \dfrac{(1-\phi_N)}{\phi_N}$

where, k_d is the sorption coefficient, ρ_N is the density of each sub layer and ϕ_N is the porosity of the each sub layer. In case under no sorption condition $(k_d \approx 0)$, the retardation factor is treated as constant. D_{0N} and U_{0N} are the initial dispersion and seepage velocity coefficients, respectively for each sub layer in a geological landfill. As we know that near the dumping area of garbage, pollutant leaches slowly underground and reaches to the aquifer layer by layer as it moves downward. The impact of dispersion and flow velocity also decreases towards the depth as contaminant concentration decreases along with the depth of the aquifer. For each sub-layer of the aquifer, we assume that each layer is partially polluted. At the input boundary, some constant type point source is injected at the uppermost level which is considered near the water table and other layers are just blow the upper most layer. The aquifer is assumed semi-infinite with constant type boundary conditions to model the system mathematically. For the discontinuity at each layer, the average value of the concentration is estimated from the previous layer at each level of the input boundary.

Initially, each sub layer is not pollutant free that means some background pollutant concentration may exist in the aquifer. A constant background concentration c_{Ni} is assigned in each layer throughout the porous medium accordingly.

$$c_N(x,t) = c_{Ni}(x,t);\ t = 0,\ x_{N-1} \le x \le x_N \tag{3}$$

At each of the intermediate boundary, an average input source concentration is assumed except at the uppermost boundary a constant input source is considered as C_0.

$$c_N(x,t) = c_{N-1}(x,t);\ t > 0,\ x = x_{N-1} \tag{4}$$

At the extreme boundary, the concentration gradient is assumed to zero in each sub layer of the domain. The outlet boundary for the intermediate layer is assigned at $x = x_N$ and in case of extreme layer the outlet boundary for the semi-infinite medium is considered at $x \to \infty$.

$$\frac{\partial c_N}{\partial x} = 0 \,;\; t > 0 \,,\; x = x_N \tag{5}$$

ANALYTICAL SOLUTION

For the solution of the proposed model, transformation in the form of new time-variable is introduced (Crank, 1979) as follows:

$$T = \int_0^t \psi(\upsilon t)\, dt \tag{6}$$

Using Eqns. (2) and (6), Eqn. (1) can be written as follows:

$$R_N \frac{\partial c_N}{\partial T} = D_{0N} \frac{\partial^2 c_N}{\partial x^2} - U_{0N} \frac{\partial c}{\partial x} - \lambda_N c_N \tag{7}$$

where, $\lambda_N = \dfrac{\lambda'_N}{\psi(\upsilon t)}$ is described as first-order decay term for each sub layer of the aquifer zone.

And the corresponding initial and boundary conditions for each sub layer can be written in new time variable as follows:

$$c_N(x,T) = c_{Ni}(x,T) \,;\; T = 0 \,,\, x_{N-1} \le x \le x_N \tag{8}$$

$$c_N(x,T) = c_{N-1}(x,T) \,;\; T > 0 \,,\; x = x_{N-1} \tag{9}$$

$$\frac{\partial c_N}{\partial x} = 0 \,;\; t > 0 \,,\; x = x_N \tag{10}$$

Now to remove the advection term from Eqn. (7), the following transformation is used:

$$c_N(x,T) = \eta_N(x,T) \exp\left\{ \frac{U_{0N}}{2D_{0N}} x - \frac{1}{R_N}\left(\frac{U_{0N}{}^2}{4D_{0N}} + \lambda_N \right) T \right\} \tag{11}$$

Case study of each sub domain

The main objective of this study is to predict the pollutant concentration behaviour in multilayer aquifer. The impact of solute transport is observed in each sub-layer at the same time period and therefore, the pollutant concentration behaviour in each sub-layer with specified length can be shown accordingly. The length of each sub-layer is considered such as for the first layer $x_0 \le x \le x_1$, for the second $x_1 \le x \le x_2$, for the third $x_2 \le x \le x_3$ and so on for the remaining layers. Here, $x_0 = 0$ is the origin at which pollutant is injected into the aquifer from the UML of the domain.

For $N = 1$, the uppermost layer (UML) of the aquifer, the proposed model is given below:

$$R_1 \frac{\partial c_1}{\partial T} = D_{01} \frac{\partial^2 c_1}{\partial x^2} - U_{01} \frac{\partial c}{\partial x} - \lambda_1 c_1 \tag{12}$$

Inlet and outlet boundary conditions given in Eqns. (8-10) can be transformed as follows:

$$c_1(x,T) = c_{1i}(x,T) \,;\; T = 0 \,,\; x_0 \le x \le x_1 \tag{13}$$

$$c_1(x,T) = c_0(x,T) \,;\; T > 0 \,,\; x = x_0 \tag{14}$$

$$\frac{\partial c_1}{\partial x} = 0\,;\ T > 0\,,\ x = x_1 \tag{15}$$

Initially, the position x_0 is assumed at the origin i.e., $x_0 = 0$ and $c_0(x,T) = C_0$ is the input initial maximum source contaminant concentration at the origin. c_{1i} is assumed as constant background concentration for the UML of an aquifer and D_{01} and U_{01} are the longitudinal dispersion and seepage flow velocity along the x-direction of the UML. The effect of the first order decay λ_1, linear sorption k_d and retardation factor R_1 are considered. The solution of the modelled problem for the Eqns. (12-15) is written as follows (Singh et al., 2014; Singh & Das, 2015):

$$c_1(x,T) = \eta_1(x,T)\exp\left\{\frac{U_{01}}{2D_{01}}x - \frac{1}{R_1}\left(\frac{U_{01}{}^2}{4D_{01}} + \lambda_1\right)T\right\} \tag{16}$$

where, $\eta_1(x,T) = C_0 * A_1 + c_{1i} * B_1 + C_0 * C_1$ (17)

As we mentioned earlier that the length of UML aquifer ranges $x_0 \le x \le x_1$, where, $x_0 = 0$ and $x_1 = L_1$ (length of the first sub-domain) and the details of A_1, B_1 and C_1 are described in appendix-1.

The solution of the second sub layer of the landfill is discussed for the domain length $x_1 \le x \le x_2$. As this region of the aquifer is not directly connected with pollutant dumping area and therefore, the input source for this region from the previous layer is arrived. We assumed some average value of concentration at the intermediate boundary as an input point source for the second layer. The model for the second sub-layer i.e., $N = 2$ at the same time period is written as follows:

$$R_2\frac{\partial c_2}{\partial T} = D_{02}\frac{\partial^2 c_2}{\partial x^2} - U_{02}\frac{\partial c_2}{\partial x} - \lambda_2 c_2 \tag{18}$$

The initial and boundary conditions for the second sub layer are taken as follows:

$$c_2(x,T) = c_{2i}(x,T)\,;\ T = 0\,,\ x_1 \le x \le x_2 \tag{19}$$

$$c_2(x,T) = c_1(x,T)\,;\ T > 0\,,\ x = x_1 \tag{20}$$

$$\frac{\partial c_2}{\partial x} = 0\,;\ T > 0\,,\ x = x_2 \tag{21}$$

All the parameter which we have taken in this section has the similar meaning as in UML of the problem. The pollutant concentration distribution is obtained for $c_2(x,T)$ with the set of Eqns. (18-21). Since this region of the aquifer lies between x_1 to x_2 so, the length of the sub-layer varies from $x_1 = L_1$ to $x_2 = L_2$. The required solution of this layer is obtained as follows:

$$c_2(x,T) = \eta_2(x,T)\exp\left\{\frac{U_{02}}{2D_{02}}x - \frac{1}{R_2}\left(\frac{U_{02}{}^2}{4D_{02}} + \lambda_2\right)T\right\} \tag{22}$$

where, $\eta_2(x,T) = \exp(-\alpha L_1)(c_1 * A_2 + c_{2i} * B_2 + c_1 * C_2)$ (23)

The details of $\eta_2(x,T)$ is shown in appendix-1 and the coefficients A_2, B_2 and C_2 mentioned in Eqn. (23) can be obtained in a similar manner as A_1, B_1 and C_1 has been obtained for the first layer. Similarly, the same can be repeated for the $3rd$, $4th$ and up to

N^{th} layer. At each intermediate sub layer, the value of input point source is acquired as the average concentration value from the previous one.

RESULT AND DISCUSSION

The pollutant concentration in semi-infinite multilayer porous medium is investigated analytically for two or three layers only. Initially, each layer of geological formation is contaminated with some constant background concentration. As we move along with the aquifer length, the contaminant concentration decreases. In each level, initial background concentration is considered in decaying form from upper to lower layer. The strength of pollutant concentration of the input source is assumed $1\,(mg/L)$ at the inlet boundary of the UML. The intermediate inlet boundary for each sub-layer is assumed as some average value of concentration from the previous layer and time is taken in years. At each layer of the aquifer, first-order decay, sorption and the other parameters data are considered in the reasonable range from the existing hydrological literature (Singh & Kumari, 2014; Gelhar et al. 1992). The flow resistance coefficient is taken $\upsilon = 1/year$. Dispersion and flow velocity are considered as varying sinusoidal form i.e., $f(\upsilon t) = 1 - \sin(\upsilon t)$. As we know that the groundwater seeps slowly with time, so for mathematical modelling of the system, very few functions are often used along with seepage flow. If we take the series expansion of such type of functions, we see that the noteworthy contribution is coming from constant source and a negligible impact is coming from the variable-dependent term. One can use constant source at the inlet boundary instead of time-dependent one. The model is applied for addressing the groundwater pollution problems. For each sub-layer of geological formulation, the sinusoidal dispersion, velocity, average porosity ϕ and bulk density ρ are considered as follows (Manger, 1963; Freeze & Cherry, 1979).

Table1. Input data for graphical illustrations for each layer of domain.

Parameters	Value
U_0	0.02 *km/ year*
D_0	0.035 $km^2/year$
λ_0	0.1/ *year*
K_d	0.15
ρ	2.19

Case analysis of result and discussion

Case1: In the proposed study, the whole aquifer is considered as a single geological formulation in a semi-infinite domain, with the porosity $\phi = 0.32$ (gravel) and initial background concentration $c_{1i} = 0.1$ mg/L. The domain of the aquifer is considered as $0 \le x \le 0.8$ km at the fixed time period *t*=2 years and *t*=3 years with the linear sorption and first order decay terms. Other required data for the graphical presentation is given in the Table 1. Here, we discussed two real-life scenarios for the pollutant migration in aquifer (1) In the presence of input source at the inlet boundary and (2) In the absence of input source at the inlet boundary.

In the presence of the input source i.e., $(C_0 = 1)$, at $x_0 = 0$, pollutant concentration is started from the peak of the input source and subsequently decreases along with the space variable x, it is represented by the blue line curve shown in the figure 2. In the absence of input source concentration i.e., $(C_0 = 0)$ is started from the origin of the domain and vanished up to initial present background contaminant concentration in the aquifer represented by the black line curve shown in the figure 2. As we know that pollutant concentration increases with increasing time, so the contaminant concentration is higher at t=3 years, than compare to t=2 years in the presence of input pollutant source. But, in the absence of input source, it reverses the scenario for the pollutant distribution under the same situations. Pollutant concentration is started from the lowest concentration value i.e., zero and is vanished up to strength of initial background concentration to the other end of the domain.

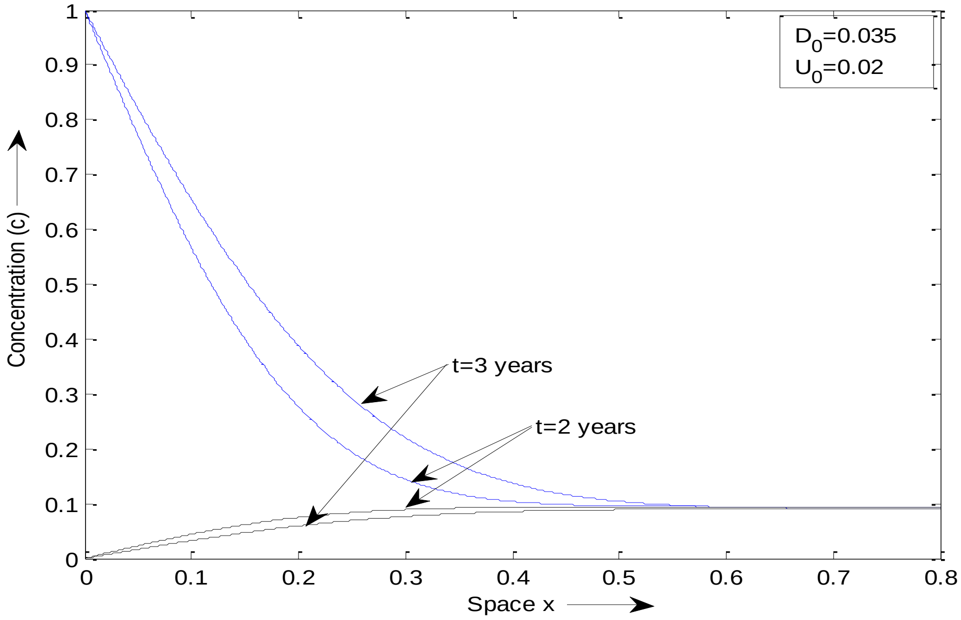

Figure 2. Pollutant concentration distribution profile for the case (1), in presence or absence of input point source at the inlet boundary of the domain.

The pollutant concentration distribution for the single-layer aquifer at fixed time t=1 year is discussed for the input source at the inlet boundary, i.e., $x_0 = 0$ due to agricultural, drainage wells or industrial activity with the fixed dispersion and different seepage velocities. Pollutant concentration decreases towards the other end of domain and is asymptotically approached to zero may be because of remedial measures taken. We observed from figure 3 that the concentration values for all four velocity profiles initially started from 1 mg/L at the inlet location of the aquifer for the domain length $0 \le x \le 0.8$ km. The concentration value attains maximum at each position for the higher velocity profile compare to low velocity in gravel medium. The rate of decrease of the pollutant concentration is faster for the low-velocity profile along the flow direction as shown in the figure 3.

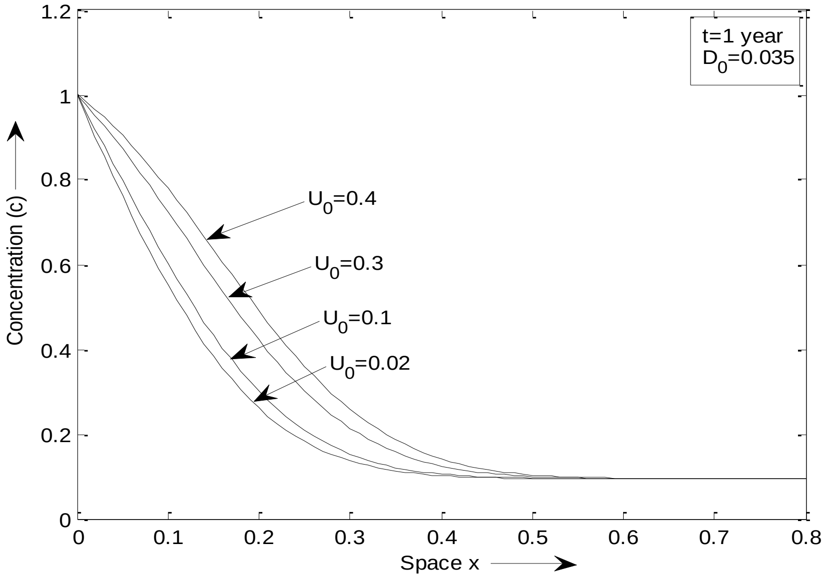

Figure 3. Pollutant concentration distribution profile for case (1) at different value of seepage flow velocity.

Case2: In this case we predicted the contaminant concentration profile for two layers of aquifer with different initial background concentrations. For the figures 4 and 5, the first layer $c_{1i} = 0.1$ mg/L and second layer $c_{2i} = 0.001$ mg/L under same geological properties are considered as proposed in Table 1. The length of the domain is considered in two range $0 \le x_1 \le 0.3$ km and $0.3 \le x_2 \le 0.8$ km for the first layer and second layer, respectively. Input point source i.e., $C_0 = 1$ is considered at the UML of the aquifer and for the other average value of concentration is taken at the inlet boundary of the lower layer.

The pollutant concentration distribution profile is shown in figure 4 for two-layers of aquifer at different time period with the same sorption and first-order decay terms as considered in Table 1, with the same porosity $\phi = 0.32$ (gravel) in both the layers. We observed that the contaminant concentration profile initially starts from the input value, i.e., at $x_0 = 0$ of the UML of the aquifer. It means that the concentration value at the inlet location of the first layer of the aquifer is highest, but the concentration value decreases when distance increases at the three particular time periods. At $t = 5$ years contaminant concentration attains higher concentration level compare to $t = 3$ years and $t = 1$ year. For both the layers, pollutant concentration is higher for an extended time period as compare to the small-time period. The rate of decrease of pollutant concentration is faster for a shorter time period towards the outlet boundary in both the layers.

The contaminant concentration profile is depicted and shown in figure 5 for the two-layers of aquifer with different porosity of each layer at the fixed time period at $t = 2$ years and the other transport parameters data is considered as in Table 1. The porosity of the upper layer (gravel) i.e., $\phi = 0.32$ and three different types of porous medium for the second layer are assumed. Figure 5 shows that contamination concentration decreases along with space variable x for the first layer and some average value of pollutant concentration is taken at the inlet of the second layer from the previous one. The pollutant concentration distribution is observed for the second layer in three different porous medium such as clay, gravel and sandstone. It is also observed that the contaminant concentration for the higher porosity (clay)

$\phi = 0.55$ is higher than that of $\phi = 0.32$ (gravel) and $\phi = 0.23$ (sandstone). The rate of increase of pollutant concentration is faster in higher porous medium than that of lower one.

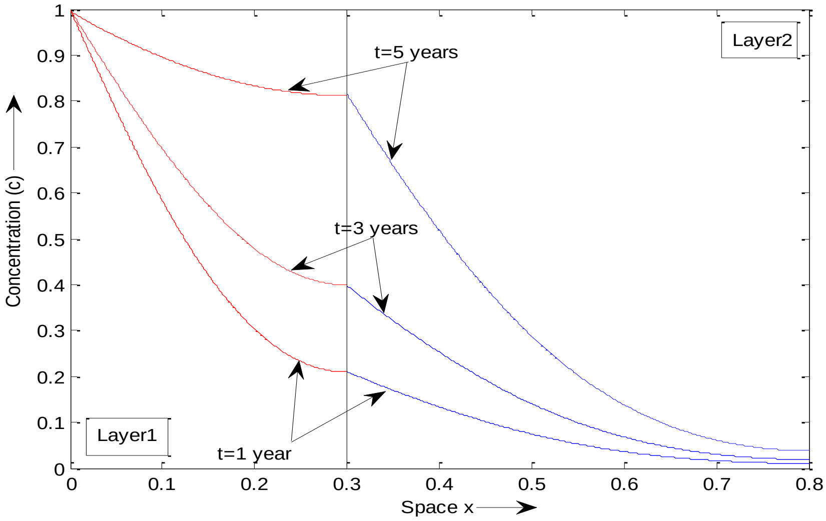

Figure 4. Pollutant concentration distribution profile for case (2) in two layer aquifer at different time period.

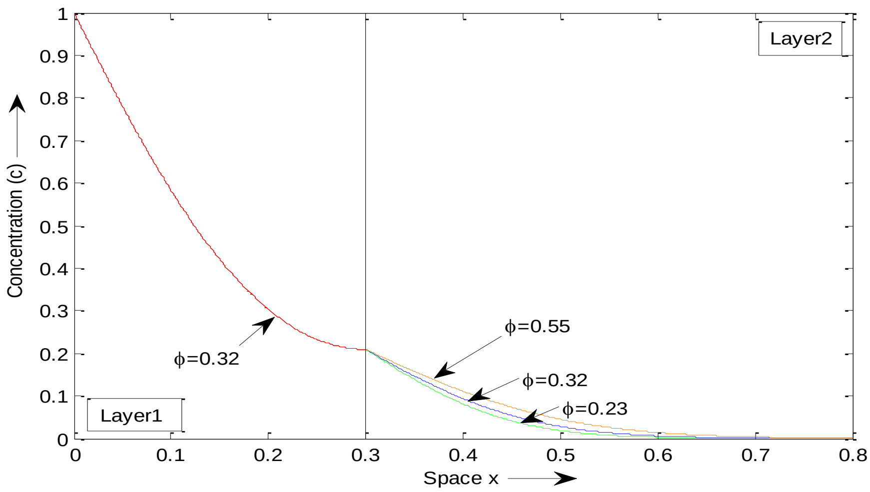

Figure 5. Pollutant concentration distribution profile for case (2), with different porosity for the second layer.

Case 3: In this case, outlet boundary of the last layer is assumed at $x \to \infty$ as aquifer domain is considered semi-infinite. Here, we studied simply three layers of aquifer in the porous medium. Length for each layer is considered as $0 \le x_1 \le 0.3$ km, $0.3 \le x_2 \le 0.6$ km and $x_3 \ge 0.6$ km, respectively and all the parameters for the graphical representations are considered according to Table 1.

The pollutant distribution profile is discussed in three layers of aquifer and shown in the figure (6). We assumed the geological formations with the same hydraulic property as described in Table 1. Initially, each layer is assumed as not solute free and therefore the background concentration of each layer is considered as $c_{1i} = 0.1, c_{2i} = 0.001, c_{3i} = 0.001$ mg/L, respectively with porosity $\phi = 0.32$ (gravel) at the fixed time period t =2 years. A sinusoidal varying velocity pattern is considered in each layer. The inlet source concentration for the UML is considered same as the previous cases, and for the intermediate layer the source concentration at the inlet boundary is assumed as some average value of concentration from its upper layer for the continuity. The three pollutant concentration patterns are shown in figure (6) and represented by red, blue and green line curves. In the first domain, contaminant concentration is maximum because of the presence of input source at $x = 0$ and it goes on decreasing from layer to layer. In the present problem the pollutant distribution goes on decreasing along the space variable. The pollutant concentration profile in the last layer shown by green line curve is almost vanished up to zero than compare to the previous layer.

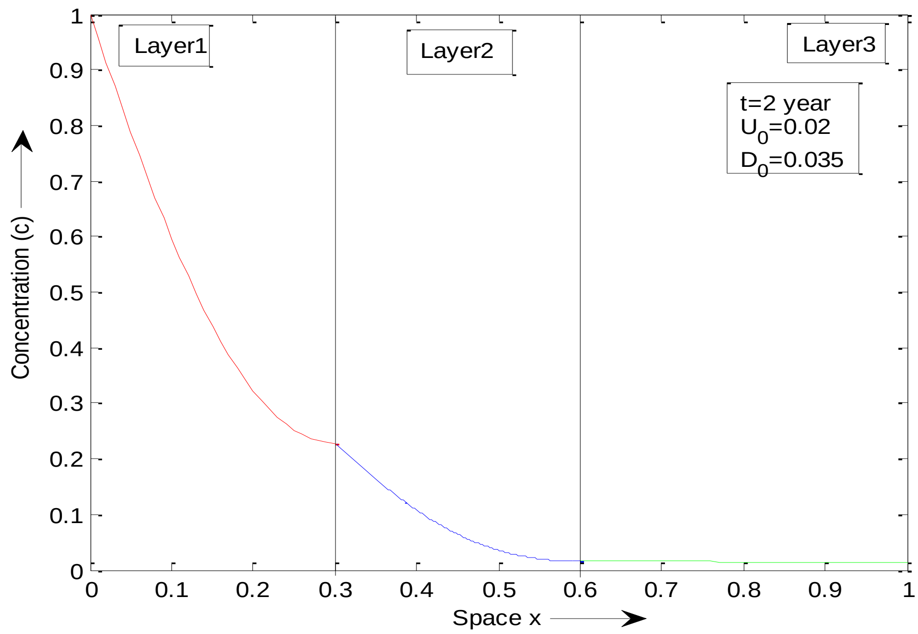

Figure 6. Pollutant concentration distribution profile for case (3), in three-layer aquifer.

The pollutant concentration distribution is predicted with respect to time at some fixed locations with the same input data as shown in Table 1 with dispersion and sinusoidal velocity pattern $(D_N = D_{0N} f(\upsilon t), U_N = U_{0N} f(\upsilon t))$. The results are discussed for the two-layer aquifer in gravel porous medium as shown in figure (7). The pollutant distribution curves are observed in time domain at $t = 2$ years. Here, we discussed two cases for the pollutant distribution in time domain at the two sets of locations $(x_1 = 0.3, x_2 = 0.5)$ km and $(x_1 = 0.5, x_2 = 0.3)$ km, respectively. The pollutant concentration level decreases on increasing the value of space variable in the both cases. Also, the concentration pattern corresponding to the location $(x_1 = 0.5, x_2 = 0.3)$ km shows the variation in concentration distribution with respect to the increasing time. Similarly, for the second location $(x_1 = 0.3, x_2 = 0.5)$ km shows the similar pattern in specified time domain. The contaminant concentration for the layer 1 and layer 2 is shown by a blue and black line curves as expressed in figure 7. The pollutant concentration for the location 1 and location 2 decreases on increasing value of the space variable and increases with respect to time variable.

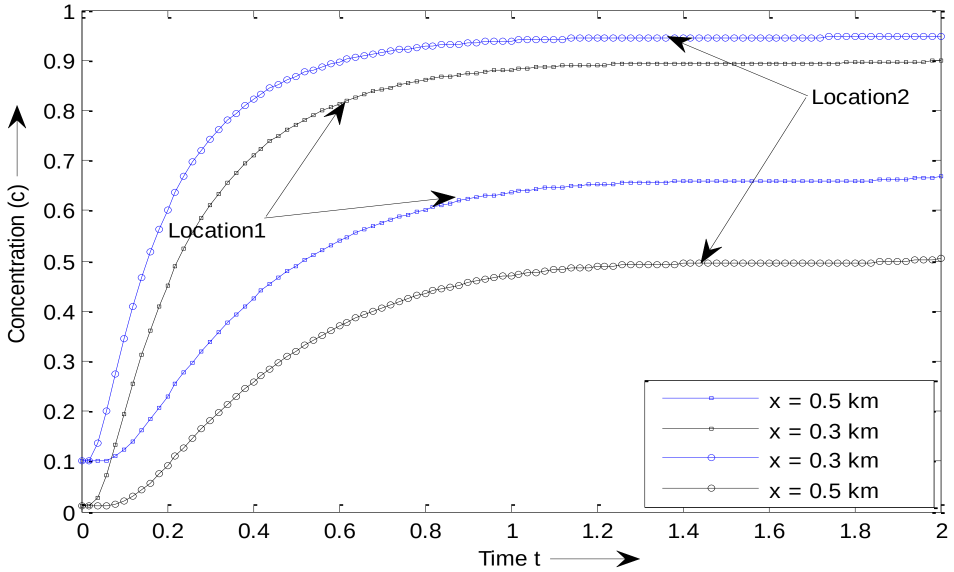

Figure 7. Pollutant concentration distribution profile at different locations in two layer aquifer.

Affirmation and authorisation of the problem

Groundwater pollution problem has been discussed extensively by many researchers in the last three to four decades with respect to solute transport modelling in a landfill. Only few studies were made in a multilayer aquifer system. In most of their problems, constant dispersion and constant seepage velocity or only the diffusion is considered for modelling. The multilayer aquifer in landfill is also discussed by many researchers experimentally. The coupled flow and advective-dispersion was simulated in multilayer leaky aquifer through a numerical transport models (Székely, 1987). A study for the transportation of salt-water in multilayer groundwater system underlying the Bangkok metropolitan area was proposed in (Gangophayay & Gupta, 1995) and in their study of the particular region, it was observed that local area basically consists of multilayer aquifer in the closed-form of clay-sand-gravel-sand-gravel. A field study of particular site location of Nigeria was carried out by (Yoshida et al., 2002) and groundwater sampling was made at a lake-side of municipal solid waste (MSV) landfill. Three different aquifers below the landfill were found and observed that all the aquifer isolated with each other. It was observed that the pollutant was exceptionally higher in the upper most aquifer than lower one. Similarly, various results were discussed regarding the solute transport modelling in multilayer aquifer (Leij & Van Ganatchen, 1995; Guerrero et al., 2013) and provided the same effect of pollutant concentration concerning space and time variable.

CONCLUSION

The study of groundwater pollutant concentration distribution is carried out for the semi-infinite multilayer aquifer which is further divided in finite number of sub layers. Laplace transform technique is adopted to obtain the solute transport behaviour in the multilayer aquifer. Pollutant transport phenomenon is studied by considering different cases for multilayer geological landfill. The pollutant concentration is decreased with the space variable in each layer of the aquifer and approach to zero at the extreme end of the aquifer. The pollutant concentration is increased for the higher porous geological material under the same dispersion and groundwater seepage flow. The change in concentration level is also observed in different time periods. The pollutant concentration in multilayer aquifer, decay pattern is remains same in each layer of geological landfill with same geological properties.

Appendix-1

At each of sub layer of aquifer, pollutant concentration can be obtained by applying the Laplace transform technique (LTT). Using Eqn. (11) in Eqns. (12-15) for $N=1$, we obtained the followings:

$$R_1 \frac{\partial \eta_1}{\partial T} = D_{01} \frac{\partial^2 \eta_1}{\partial x^2} \tag{24}$$

$$\eta_1(x,T) = c_{1i} \exp(-\alpha x);\ T=0,\ 0 \le x \le L_1 \tag{25}$$

$$\eta_1(x,T) = C_0 \exp(\beta T);\ T>0,\ x=0 \tag{26}$$

$$\frac{\partial \eta_1}{\partial x} + \alpha \eta_1 = 0;\ T>0,\ x=L_1 \tag{27}$$

where, $\alpha = \frac{U_{01}}{2D_{01}}$ and $\beta = \frac{1}{R_1}\left(\frac{U_{01}{}^2}{4D_{01}} + \lambda_1\right)$.

The transformed Eqns. (24-27) can be written as follows:

$$\overline{\eta_1} = c_1 \exp\left(\sqrt{\frac{PR}{D}}x\right) + c_2 \exp\left(-\sqrt{\frac{PR}{D}}x\right) + c_{1i}\frac{\exp(-\alpha\xi)}{P-m^2} \tag{28}$$

$$\overline{\eta_1}(x,P) = \frac{C_0}{P-\beta};\ x=0,\ P>0 \tag{29}$$

$$\frac{\partial \overline{\eta_1}}{\partial x} + \alpha \overline{\eta_1} = 0;\ P>0,\ x=L_1 \tag{30}$$

where, P is the transform time variable in Laplacian domain. The solution for the layer 1 in Laplacian domain can be obtained as follows:

$$\overline{\eta_1}(x,p) = \begin{bmatrix} \left(\frac{C_0}{p-\beta} - \frac{c_{1i}}{p-m^2}\right)\left\{1-2\alpha\Big/\left(\sqrt{\frac{pR}{D}}+\alpha\right)\right\} \times \exp\left\{-\sqrt{\frac{pR}{D}}(2L_1-x)\right\} \\ -\left[\frac{C_0}{p-\beta} - \frac{c_{1i}}{p-m^2}\right] \times \exp\left\{-\sqrt{\frac{pR}{D}}(2L_1+x)\right\} \\ -\left[\frac{C_0}{p-\beta} - \frac{c_{1i}}{p-m^2}\right] \times \left\{1-2\alpha\Big/\left(\sqrt{\frac{pR}{D}}+\alpha\right)\right\}^2 \exp\left\{-\sqrt{\frac{pR}{D}}(4L_1-x)\right\} \\ +\left[\frac{C_0}{p-\beta} - \frac{c_{1i}}{p-m^2}\right]\exp\left\{-\sqrt{\frac{pR}{D}}x\right\} + \frac{c_{1i}\exp(-\alpha x)}{p-m^2} \end{bmatrix} \tag{31}$$

Furthermore, by taking the inverse Laplace transform of Eqn. (31) we obtained the value of η_1 given in Eqn. (17) where A_1, B_1 and C_1 are given as follows:

$$A_1=\left[\begin{array}{l} 0.5\left\{F_1(Z_1,T)+F_2(Z_1,T)\right\}-(2m)\left\{\begin{array}{l}G_1(Z_1,T)-G_2(Z_1,T)\\ +G_3(Z_1,T)\end{array}\right\}- \\ \left(\begin{array}{l}0.5\left\{F_1(Z_1,T)+F_2(Z_1,T)\right\}\\ -(2m)\left\{\begin{array}{l}G_1(Z_2,T)-G_2(Z_2,T)\\ +G_3(Z_2,T)\end{array}\right\}\end{array}\right)-\left(\begin{array}{l}0.5\left\{F_1(Z_3,T)+F_2(Z_3,T)\right\}\\ -(4m)\left\{\begin{array}{l}G_1(Z_3,T)-G_2(Z_3,T)\\ +G_3(Z_3,T)\end{array}\right\}+\\ \left(\dfrac{4\alpha^2 D_{01}}{R_1}\right)\left\{\begin{array}{l}H_1(Z_3,T)+H_2(Z_3,T)\\ -H_3(Z_3,T)-H_4(Z_3,T)\end{array}\right\}\end{array}\right)\end{array}\right] \tag{32}$$

$$B_1=\left[\begin{array}{l} -0.5\left\{F_3(Z_1,T)+F_4(Z_1,T)\right\} \\ +(2m)\left\{\begin{array}{l}G_4(Z_1,T)-G_5(Z_1,T)\\ +G_6(Z_1,T)\end{array}\right\}+\left(\begin{array}{l}0.5\left\{F_3(Z_1,T)+F_4(Z_1,T)\right\}\\ -(4m)\left\{\begin{array}{l}G_4(Z_1,T)-G_5(Z_1,T)\\ +G_6(2L+2H+x,T)\end{array}\right\}\end{array}\right) \\ +\left(\begin{array}{l}0.5\left\{F_3(Z_3,T)+F_4(Z_3,T)\right\}\\ +(2m)\left\{\begin{array}{l}G_4(Z_3,T)-G_5(Z_3,T)\\ +G_6(Z_3,T)\end{array}\right\}\end{array}\right)+\left(\dfrac{4\alpha^2 D}{R}\right)\left\{\begin{array}{l}I_1(Z_3,T)-I_2(Z_3,T)\\ +I_3(4L+4H-x,T)\end{array}\right\}\end{array}\right] \tag{33}$$

$$C_1=\left[\begin{array}{l}0.5C_0\left\{F_1(Z_4,T)+F_2(Z_4,T)\right\}-\\ 0.5c_{1i}\left\{F_3(Z_4,T)+F_4(Z_4,T)\right\}+c_{1i}\exp(-\alpha x)\exp\left(m^2T\right)\end{array}\right] \tag{34}$$

The other parameters involved in Eqns. (31-34) are given as follows:
$(Z_1=2L_1-x);(Z_2=2L_1+x);(Z_3=4L_1-x);(Z_4=x)$

$$F_1(x,T)=\exp\left(\beta T-\sqrt{\frac{\beta R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x-\sqrt{\beta T}\right)$$

$$F_2(x,T)=\exp\left(\beta T+\sqrt{\frac{\beta R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+\sqrt{\beta T}\right)$$

$$F_3(x,T)=\exp\left(m^2T-m\sqrt{\frac{R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x-m\sqrt{T}\right)$$

$$F_4(x,T)=\exp\left(\beta T+\sqrt{\frac{\beta R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+m\sqrt{T}\right)$$

$$G_1(x,T)=\frac{1}{2\left(\sqrt{\beta}+m\right)}\exp\left(\beta T-\sqrt{\frac{\beta R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x-\sqrt{\beta T}\right)$$

$$G_2(x,T)=\frac{1}{2\left(\sqrt{\beta}-m\right)}\exp\left(\beta T+\sqrt{\frac{\beta R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+\sqrt{\beta T}\right)$$

$$G_3(x,T)=\frac{m}{\left(\beta-m^2\right)}\exp\left(m^2T+m\sqrt{\frac{R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+m\sqrt{T}\right)$$

$$G_4(x,T)=\sqrt{\frac{T}{\pi}}\exp\left(-\frac{R_1}{4D_{01}T}x^2\right)$$

$$G_5(x,T)=\left(\frac{1}{4m}\right)\exp\left(m^2T-m\sqrt{\frac{R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x-m\sqrt{T}\right)$$

$$G_6(x,T)=\left(\frac{1}{4m}\right)\left(1+2m\sqrt{\frac{R_1}{D_{01}}}x+4m^2T\right)\exp\left(m^2T+m\sqrt{\frac{R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+m\sqrt{T}\right)$$

$$H_1(x,T)=\frac{1}{2\left(\sqrt{\beta}+m\right)^2}\exp\left(\beta T-\sqrt{\frac{\beta R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x-\sqrt{\beta T}\right)$$

$$H_2(x,T)=\frac{1}{2\left(\sqrt{\beta}-m\right)^2}\exp\left(\beta T+\sqrt{\frac{\beta R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+\sqrt{\beta T}\right)$$

$$H_3(x,T)=\frac{\beta+m^2}{\left(\beta-m^2\right)^2}\exp\left(m^2T+m\sqrt{\frac{R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+m\sqrt{T}\right)$$

$$H_4(x,T)=\frac{m}{\left(\beta-m^2\right)}\exp\left(m^2T+m\sqrt{\frac{R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+m\sqrt{T}\right)\left(2mT+\sqrt{\frac{R_1}{D_{01}}}x\right)$$
$$-\frac{2m}{\left(\beta-m^2\right)}\sqrt{\frac{T}{\pi}}\exp\left(m\sqrt{\frac{R_1}{D_{01}}}x\right)$$

$$I_1(x,T)=\frac{1}{8m^2}\exp\left(m^2T-m\sqrt{\frac{R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x-m\sqrt{T}\right)$$

$$I_2(x,T)=\frac{1}{2m}\sqrt{\frac{T}{\pi}}\left(1+2m\sqrt{\frac{R_1}{D_{01}}}x+2m^2T\right)\exp\left(-\frac{R_1}{4D_{01}T}x^2\right)$$

$$I_3(x,T)=\frac{1}{8m^2}\left\{-1+2m\sqrt{\frac{R_1}{D_{01}}}x+8m^2T+2m^2\left(\sqrt{\frac{R_1}{D_{01}}}x+2m^2T\right)^2\right\}$$
$$\times\exp\left(m^2T+m\sqrt{\frac{R_1}{D_{01}}}x\right)erfc\left(\sqrt{\frac{R_1}{4D_{01}T}}x+m\sqrt{T}\right)$$

and $m = \frac{U_{01}}{2\sqrt{D_{01}R_1}}$.

On substituting the value of A_1, B_1 and C_1 in Eqn. (17) we obtained the required concentration values for the UML. Similarly, one can predict the contaminant concentration for the second sub-layer as the domain of this region lies between $L_1 \le x \le L_2$. Eqns. (18-21) can be further obtained by applying the similar technique as discussed above for the previous layer and the input source for the second layer will be average pollutant concentration at the intermediate boundary from the UML to maintain the continuity. The value of the parameters A_2, B_2 and C_2 in Eqn. (23) can be further obtained in similar manner as in Eqn. (17). The intermediate value for the coefficients Z_1, Z_2 and Z_3 for the second layer will replace in Eqns. (28-30) as

$$\left(Z_1 = 2L_2 - L_1 - x\right);\left(Z_2 = 2L_2 - 3L_1 + x\right);\left(Z_3 = 4L_2 - 3L_1 - x\right);\left(Z_4 = x - L_1\right)$$

By substituting all these values in Eqn. (22) we obtained the solution for the second sub layer. On proceeding in a similar manner, we may able to find the solution for $3rd$, $4th \ldots$ up to N^{th} sub layer as well.

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CONFLICT OF INTEREST

The authors declare that there is not any conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/ or falsification, double publication and/or submission, and redundancy has been completely observed by the authors.

LIFE SCIENCE REPORTING

No life science threat was practiced in this research.

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