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Fuzzy Synthetic Evaluation for classifying Groundwater quality for irrigation in the parts of Tumkur district, Karnataka: Insights of uncertainty management at class boundary condition

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ABSTRACT

Sustainability of irrigated agriculture is based on the efficient management of quantity and quality of water resources. Water Quality Indices used to assess the suitability of irrigation water, however, often consist of uncertainties arise near the class boundaries. Hence, the objective of the present study is to classify the groundwater for irrigation purpose in Tumkur district, Karnataka, India, using Fuzzy comprehensive evaluation approach for crisp classification. The methodology of this study includes collection of 104 groundwater samples, assessment of hydrogeochemistry, and classification of groundwater by conventional and Fuzzy-logic techniques. Hydrogeochemistry by Piper plot indicates mixed Na-Ca-HCO3 type and Gibbs plot indicates the influence of rock-water interactions. The water classification by conventional irrigation indices such as Electrical Conductivity, Sodium Absorption Ratio, Kelly Index, Percentage Sodium, Residual Sodium Carbonate and Magnesium Hazard showed that 2%, 0%, 86.5%, 40%, 25% (post monsoon) and 4%, 2%, 81%, 38.5%, 4% and 19.2% (pre-monsoon) of groundwater samples were not suitable, respectively. As various indices indicated dissimilar results, an integrated conventional index was evaluated by Fuzzy synthetic evaluation technique based on the Maximum Principle Membership and Fuzzy Class Ratio (FCR) and it showed 3.8 % and 0.98% of samples were classified as Not suitable (N), respectively. However, FCR method was found to be effective in dealing variation in fuzzy boundary conditions and it showed 0.98%, 1.96%, 1.96%, 1.96% samples as not suitable at 5%, 10%, 15% and 20% of degree of variation near class boundaries, respectively.

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INTRODUCTION

Agriculture plays a pivotal role in ensuring food security and driving India's economic development. Agriculture serves as the primary occupation and livelihood source particularly for the rural populace. The Green Revolution has led to a notable increase in the net irrigated area, predominantly relying on both surface and groundwater resources (Ambika et al., 2016). It is apparent from the Fig. S1 that, during the period between 1950 and 2018, the extent of surface water based net irrigation area is almost remained constant, whereas the extent of groundwater based net irrigation area has amplified by more than six-fold. The dependency on the groundwater is high in India due to torrential rainfall, lack of surface water management, inadequate irrigation infrastructures and vast array of non-perennial river system. The overexploitation of irrigation blocks is rising at an unsustainable rate of 5.5% per year, indicating an impending crisis in groundwater irrigation in India that necessitates immediate and thorough

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intervention (Gandhi & Bhamoriya, 2011; Zaveri et al., 2016). Thus, in this situation, evaluating the groundwater quality, extensively utilized for irrigation, becomes vital.

The quality of groundwater is influenced by its hydrogeochemical reactions with bedrock or soil in contact, quality and quantity of recharging water, and other anthropogenic activities (irrigation return flow, discharge of untreated or partially treated industrial effluent and sewage) (Maghrebi et al.,2021). The major water quality issues in irrigated water that can damage the crop growth and soil's characteristics are Salinity, Sodicity and accumulation of certain toxic ions (Ayers & Westcot, 1985; Ayars & Tanji, 1999). Various methods and approaches have been adopted to interpret and understand the complex information of the quality of irrigation water. The indices such as Electrical Conductivity (EC), Sodium Absorption Ratio (SAR), Percentage of Sodium (Na%), Permeability index (PI), Magnesium Hazard (MH), Residual Sodium Carbonate (RSC), Soluble Sodium Percentage (SSP), Permeability Index (PI) are the major parameters considered for the evaluation irrigation water quality (Mia et al.,2023; Gautam et al. 2015). Further, graphical methods such as United States Soil Laboratory (USSL) classification system (Richards, 1954); Wilcox system (Wilcox, 1955) were also used to assess the irrigation water quality. Water Quality Indices (WQIs) computed through aggregating sub-indices and assigning weights, offer a comprehensive method for water classification, with advanced techniques incorporating Remote Sensing (RS), Geographical Information System (GIS) and Multi Criteria Decision Making (MCDM) (Zahedi, 2017; El Behairy et al., 2021; Sutradhar & Mondal, 2021).

Nevertheless, though these indices have an advantage of easy estimation of water classification through the single valued index, in some cases, especially when handling environment and experimental uncertainties, it results in imprecise classification. Each component of water quality index, involved with various inherent uncertainties, such as indistinct information about the set of parameters, emergence of biases during the sub-indices' computation and weight assignment, ambiguity and vagueness in the output classification for the aggregated values nearby boundaries. Additionally, conventional classifications through the index or graphical appraches, often leads to binary classifications, introducing challenges in cases where samples exhibit near-equal probabilities of belonging to different classes. This imprecision, especially in the context of agricultural irrigation, could have impact on crop yield and farmers' economic conditions. Therefore, a sophisticated decision-making approach is necessary to address this limitation.

Fuzzy logic emerges as a mathematical solution pioneered by Zadeh (1965), designed to navigate uncertainty, ambiguity, and non-linearity. Numerous studies have used fuzzy techniques to handle complexities in irrigation water evaluation. Mirabbasi et al. (2008) integrated fuzzy inference systems to address uncertainties related to boundary values, while Alavi et al. (2010) compared two FIS systems for USSL classification. Chidambaram et al. (2022) incorporated fuzzy logic and GIS for irrigation water classification, and Dhaoui et al. (2022) employed FIS to develop a Water Quality Index assessing groundwater quality for irrigation using multiple parameters. Previous studies have emphasized the efficacy of fuzzy logic techniques in addressing gradual variations between classes. However, there remains a notable gap in considering a distinctive indicator to precisely specify the level or degree of variation at boundaries. This aspect is crucial because the determination of the „type and level of variations" relies on evaluator or expert knowledge. Without a well-defined indicator, there is a risk of introducing errors or imprecisions in water quality results, especially when assessing variations above acceptable limits. Therefore, the present study, introduces a fuzzy logic-based index method by considering the specific degree of variation at the boundary line, which could improve the quality of irrigation water classifications. The objectives of this study are (i) to study the hydrogeochemical characteristics of groundwater (ii) to assess groundwater suitability for irrigation purpose using a fuzzy logic-based water quality index.

MATERIALS AND METHODS

Study area

The general details of the study area are tabulated in Table. S1. The climate of the region is semi-arid tropical, which receives an average rainfall around 700 mm/yr. Both the sub-districts, situated in between latitudes of 13° 19' 4.8" N and 13° 55' 48" N, and longitudes of 77° 0' 28.8" E and 77° 28' 15.6" E (Fig. 1) and covering a geographical area of 1.83 thousand hectares out of which 63% of the area is agricultural area.

Agriculture is the primary occupation in this region, focusing on crops such as Maize, Ragi, Paddy, Groundnut, Horsegram, and Redgram. While Jayamangali and Suvarnamukhi rivers traverse the area, they are non-perennial, so agricultural water needs rely heavily on rainfall during monsoon. Groundwater is extensively utilized during non-monsoon seasons. Decadal water level data (2008-2018) indicates a 0-2 m rise during monsoons but a >4 m fall in summers (Dynamic Groundwater Resources of Karnataka, 2020), highlighting severe depletion. Additionally, Table. S1 reveals, this study region exhibits high fertilizer usage compared to neighboring sub-districts (Economic Survey report, 2021), implying vulnerability to groundwater contamination. Increase rates of groundwater withdrawal leads to issues, such as declline of groundwater table, reduction of yield, and deterioration of water quality, influenced by natural factors (climate and geology etc), and human factors (agriculture activities), that can have devastating effects on regional food security (Central Ground Water Board, 2017, Noori et al., 2023). Therefore, a proper groundwater management is essential to protect water quality, and ensure its sustainable utilization in this region.

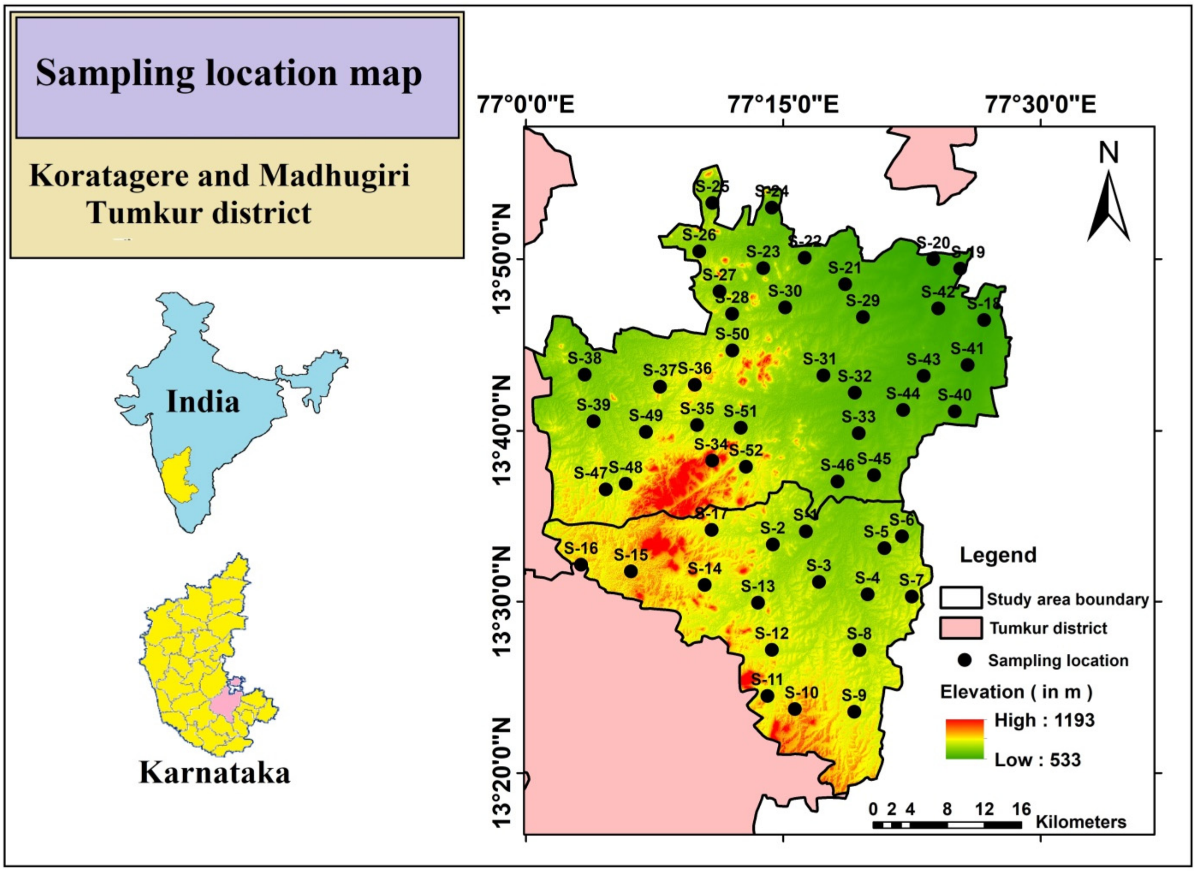

Fig. 1. Map depicting the sampling locations in the study area

Table 1. Details of the parameters and the criteria of conventional irrigation water classification

Parameters	Expressions	Criteria				Reference
		HS	S	NS	HNS	
Electrical Conductivity (EC) (in µS/cm)	-	<1500	1500-3000	3000-6000	>6000	BIS (1986)
Sodium Adsorption Ratio (SAR)	Na/((Ca+Mg)/2))^0.5	<10	10-18	18-26	>26	Richards (1954); BIS (1986)
Kelly Index (KI)	Na/(Ca+Mg)	-	<1	>1	-	Kelly (1940)
Sodium Percentage (%Na)	Na+K/(Ca+Mg+ Na+K)	>40	40-60	60-80	>80	Wilcox (1948)
Residual Sodium Carbonate (RSC) (in mg/l)	(CO_3+ HCO_3) - (Ca + Mg)	<1.5	1.5-3	3-6	>6	Eaton (1950); BIS (1986)
Magnesium Hazard (MH)	Mg/(Ca+Mg)	-	<50	>50	-	Paliwal (1972)

Note : HS- Highly Suitable, S- Suitable, NS- Not Suitable, HNS- Highly Not Suitable

Sampling and Data Analysis

Groundwater samples were collected from bore wells in October-November 2018 and March 2020, representing post-monsoon and pre-monsoon seasons, respectively. A total of 104 samples (52 from each season) were collected in one-liter polypropylene bottles rinsed with double-distilled water. pH, EC, TDS, Temperature, DO, and Salinity were measured using a YSI multi- parameter kit immediately after sampling. Major cations ($Ca^{2+,}$ Na^+, K^+) were analyzed using Flame photometry, while major anions (Cl^-, F^-, SO_4^{2-}, NO_3^-) were measured using ion-selective electrode and UV-visible spectrophotometer. Standard APHA (2012) procedures were followed. TA and TH were estimated by titration, and HCO_3^- empirically. Data accuracy was validated using Ion Balance Error (Domenico & Schwartz, 1990) demonstrating compliance within the ±10 limit. Hydrogeochemical characteristics were analyzed using Piper and Gibbs diagrams. Further, Fig. S2 outlines the methodology carried out in this study.

Irrigation water quality classification

The characteristics of irrigation water can be determined by using various parameters, such as Electrical conductivity (EC), Sodium adsorption ratio (SAR), Kelly Index (KI), Percentage of Sodium (Na%) Residual Sodium Carbonate (RSC) and Magnesium Hazard (MH). The formulae for calculations and their criteria for classifying water is given the Table. 1.

Fuzzy Comprehensive Assessment

Fuzzy comprehensive evaluation is one of the effective methods for groundwater quality assessment. Studies have utilized this apprach to evalaute groundwater quality (Hao et al., 2012, Zhang et al., 2023), emphasizing its ability to manage variability and inherent uncertainties in data, and environmental complexities. In this study as well, a water quality index is developed using the same approach, with procedural steps detailed as follows.

Step 1: Establishment of parameter sets and determination of standard values

The suitable parameters (*n* numbers) that affects the water quality are represented as a set of function as follows (Eq.1).

$$\phi = \{ \phi_1, \phi_2, \phi_3 \dots \phi_n) \quad \text{Eq.1}$$

Further, the water classification system for each parameter, consist of different classes (C) defined between limit values (λ) based on the standard guideline decision criteria are represented by set functions as given in Eq. 2 and Eq. 3 respectively.

$$\lambda = \{\lambda_{0,} \lambda_{1,} \lambda_2, \lambda_3 \ldots \lambda_{(m),} \lambda_{(m+1)}\} \qquad \text{Eq.2}$$

$$C= \{C_{1,} C_2, C_3 \ldots C_{(m+1)}\} \qquad \text{Eq.3}$$

Where, m is the number of partitioner (i.e. limit values used to derive the classes). Whereas λ_0 and $\lambda_{(m+1)}$ are the lower and upper extreme bounds of the classification system. As a general rule, for 'm' number of partitioner, there will be 'm+1' number of classes and the class interval value is calculated as $C_{(m)} = (\lambda_{(m)}-\lambda_{(m-1)})$. In this study, for the evaluation of classes for irrigation water, the criteria limits given in the Table. 1 have been considered.

Step 2: Construction of Fuzzy Membership Matrix

Fuzzy logic can describe uncertainties of the measured values when approaching near the boundary limits with the help of membership function (*mf*) concept, which normally denoted as μ. The association of each element in a fuzzy set defined by *mf* values between 0 and 1. The membership function value of fuzzy set can be expressed as follows Eq. 4.

$$\mu_A(x) = \{\mu_A(x), x \in U, \mu_A(x)) \in [0, 1]\} \qquad \text{Eq.4}$$

Where, x is an element of fuzzy set A and belongs the universe of discourse U. In this study, the crisp boundary is represented as fuzzy boundary as shown in the Fig. 2. The structure of fuzzy boundary is consisting of three point locations; one is the boundary limit point and other two points in the anterior and posterior positions to it, denoted as λ_m, λ_m^+ and λ_m^-, respectively. The points λ_m^- and λ_m^+ represented the values of anterior and posterior points adjacent to the boundary limit value λ_m obtained for a given degree of variation at given boundary limit point (λ_m) and is denoted as δ (given in percentage fractions) using Eq.5.

$$\lambda_m^- = \lambda_m - \delta.min\,[(\lambda_m - \lambda_{m-1}),(\lambda_{m+1} - \lambda_m)] \qquad \text{Eq.5}$$

$$\lambda_m^+ = \lambda_m + \delta.min\,[(\lambda_m - \lambda_{m-1}),(\lambda_{m+1} - \lambda_m)] \qquad \text{Eq.6}$$

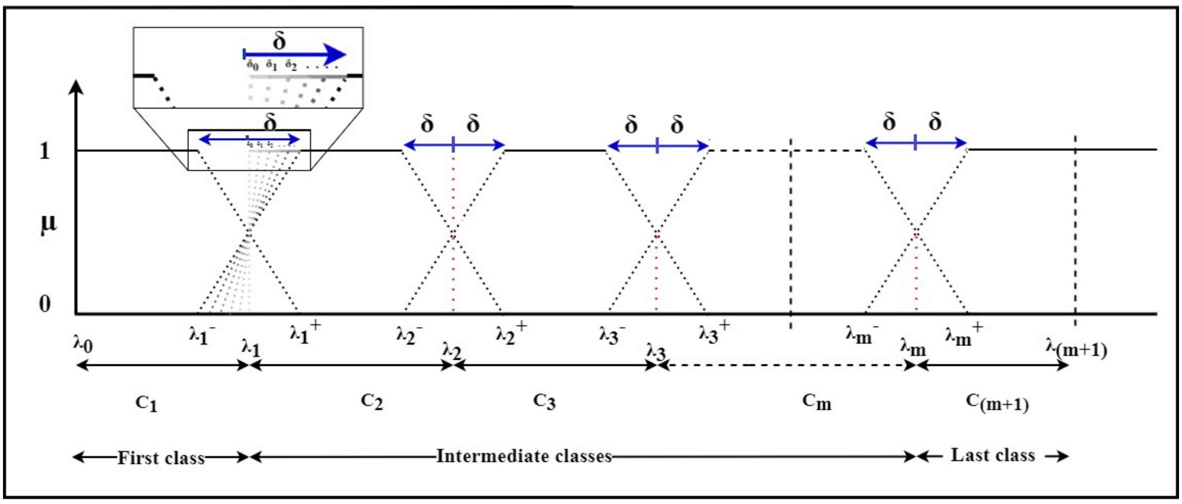

Fig. 2. Conceptual details of the boundary condition near the class border values

The values of (λ_m - λ_{m-1}) and (λ_{m+1} - λ_m) are the class intervals of adjacent classes (denoted as C_m and C_{m+1}) at that boundary limit value λ_m. In the space between these limits, the gradual variation can be demonstrated more efficiently using trapezoidal *mf* compared to others *mf*s (triangular, Gaussian, sigmoidal etc.), therefore, it has been adapted in this study. This approach introduces a notion of crossover of two adjacent trapezoidal *mf*s at its common boundary point as given in the Fig. 2 such that $\mu(\lambda_m) = 0.5$. The *min* function in the Eq.5 and 6 was used to minimize the uncertainties evolved due to uneven interval values. This approach captures the fuzziness associated in the boundary system among the parametric values, which is reflected by values of degree of membership ($\mu(x)$). Overall, the membership values of any class between any two limit values of set λ is denoted as $(\mu(x))_\lambda$ is computed from Eq.7 to 9. Since, the lowest and highest classes are associated with the extreme boundaries i.e. lower and upper bound; their corresponding values of $(\mu(x))_\lambda$ are computed using Eq.7 and Eq.8. Whereas $(\mu(x))_\lambda$ values for intermediate classes are determined using Eq.9.

$$\mu(x)_{lowest\ class} = \begin{cases} 1 & when\ x \leq \lambda^-_{(m)} \\ 0 & when\ x \geq \lambda^+_{(m)} \\ \frac{\lambda^-_{(m)} - x}{\lambda^+_{(m)} - \lambda^-_{(m)}} & when\ \lambda^-_{(m)} < x < \lambda^+_{(m)} \end{cases} \quad \text{Eq.7}$$

$$\mu(x)_{highest\ class} = \begin{cases} 0 & when\ x \leq \lambda^-_{(m)} \\ 1 & when\ x \geq \lambda^+_{(m)} \\ \frac{x - \lambda^-_{(m)}}{\lambda^+_{(m)} - \lambda^-_{(m)}} & when\ \lambda^-_{(m)} < x < \lambda^+_{(m)} \end{cases} \quad \text{Eq.8}$$

$$\mu(x)_{intermediate\ class} = \begin{cases} 1 & when\ \lambda^+_{(m)} < x < \lambda^-_{(m+1)} \\ 0 & when\ x \leq \lambda^+_{(m)}\ and\ x \geq \lambda^+_{(m+1)} \\ \frac{x - \lambda^-_{(m)}}{\lambda^+_{(m)} - \lambda^-_{(m)}} & when\ \lambda^-_{(m)} < x < \lambda^+_{(m)} \\ \frac{\lambda^-_{(m+1)} - x}{\lambda^+_{(m+1)} - \lambda^-_{(m+1)}} & when\ \lambda^-_{(m+1)} < x < \lambda^+_{(m+1)} \end{cases} \quad \text{Eq.9}$$

The obtained membership values for j[th] parameter under i[th] classification system for "S" number of analyzed samples, are represented in a fuzzy membership matrix denoted as λ_s (Eq.10.)

$$\lambda_s = \begin{bmatrix} \mu(x)_{11} & \mu(x)_{12} & \dots & \mu(x)_{1n} \\ \mu(x)_{21} & \mu(x)_{22} & \dots & \mu(x)_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu(x)_{C1} & \mu(x)_{C2} & \dots & \mu(x)_{Cn} \end{bmatrix} \quad \text{Eq.10}$$

Step 3: Weightage determination

The weightage factors of various parameters represent the degree of effect of parameters on the water quality. To balance both the subjective expertise and objective entropy details of parameter values, an integrated-weightage method is employed in this study as it is more desirable in weightage computations. The integrated-weight (ω_{int}) is determined by following

Eq.11. (Refer Supplementary Section 1).

$$\omega_{int} = \frac{\omega_{obj} \cdot \omega_{sub}}{\sum_{i=1}^{n} \omega_{obj} \cdot \omega_{sub}} \qquad \text{Eq. 11}$$

Step 4: Fuzzy algorithm

In this step, the irrigation classification of water is obtained, which is represented by a degree of each classes, expressed as a fuzzy matrix F = {F_1, F_2, F_3…., F_n}. To achieve this, fuzzy membership matrix is to be multiplied with a weight matrix as given in the Eq.12.

$$F = \lambda_s . \omega$$

$$\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} = \begin{bmatrix} \mu(x)_{11} & \mu(x)_{12} & \dots & \mu(x)_{1n} \\ \mu(x)_{21} & \mu(x)_{22} & \dots & \mu(x)_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu(x)_{C1} & \mu(x)_{C2} & \dots & \mu(x)_{Cn} \end{bmatrix} . [\omega_1, \omega_2, \omega_3, .. \omega_n] \qquad \text{Eq.12}$$

Where, λ_S is the fuzzy membership matrix obtained in the previous step and ω is the weight matrix given as

$$\omega = [\omega_1, \omega_2, \omega_3, .. \omega_n] \qquad \text{Eq.13.}$$

$$where, \sum_{1}^{n} \omega = 1$$

In this weight matrix (Eq.19) each element indicates the weight of the parameter of interest. Here, the weightage values determined using the integration approach (Eq.11) is adapted.

Fuzzy Based Classifications of Irrigation Water Quality

In the final stage of developing a water quality index for irrigation suitability, a fuzzy synthetic evaluation approach that effectively capture the interaction between considered factors and provides more accurate results (Zhang et al., 2019) was employed using two calculation methods based on maximum membership function and fuzziness measures. The resulting index values were then classified into Highly Suitable (HS), Suitable (S), Not suitable (NS) and Highly Not Suitable (HNS) classes.

Principle of maximum membership function-based classification (Method-1)

This method is based on principle of maximum membership function, one of the widely utilized method for determining the final classification in the fuzzy synthetic evaluation. In this method, the fuzzy based classification about the sample is obtained by considering the element in the fuzzy matrix that has highest degree of membership i.e. max {$F_{1...m}$}(Civanlar & Trussell, 1986). For the water classification, this method considers maximum value from the set {$F_{HS,}$ $F_{S,}$ $F_{NS,}$ $F_{HNS,}$}.

Fuzzy Class Ratio (FCR) (Method-2)

In this method, the fuzzy index values were obtained from the considering the fuzzy measures of elements in fuzzy matrix, represented as Fuzzy Class Ratio (FCR) in Eq.14.

$$\text{Fuzzy Class Ratio (FCR)} = \frac{F(Suitable) + F(highly\ suitbale)}{F(Not\ Suitable) + F(highly\ \ not\ suitbale)} \qquad \text{Eq.14}$$

The FCR values ranges varies from 0 to 100, based on which a system of water classification was constructed. The water classification consists of, four classes, Highly Suitable (FCR > 3), Suitable (1> FCR>3), Not suitable (0.3 > FCR>1),) and Highly Not Suitable (FCR < 0.3).

RESULTS AND DISCUSSION

General Hydrogeochemistry

Piper trilinear diagram (Piper, 1944) is plotted to interpret the hydrogeochemical characteristics of groundwater in the study area (Fig. 3a). The results revealed that the groundwater types in this region are dominated by mixed Na-Ca-HCO_3 water type (54% samples during both the seasons) followed by Na-Cl (25 % samples in post monsoon and 33 % samples in pre-monsoon) and Ca-HCO_3 (21 % samples in post monsoon and 13 % samples in pre-monsoon). It is observed that, alkaline (Na^+ and K^+) is dominating over the alkalis (Ca^{2+} and Mg^{2+}) among cations whereas HCO_3 dominating over other anions. The presence of these ions in groundwater is due to the dissolution underlying rocks or soil and other anthropogenic activities. Further, Gibbs diagram (Gibbs, 1970) is plotted to understand the influence of Rock–water interaction, Evaporation - Crystallization, Precipitation on the geochemistry of groundwater (Fig. 3 b-c). It is observed that most of the groundwater samples are influenced by of Rock–water interaction during both the pre and post monsoon seasons.

Irrigation Water Quality

Evaluation of suitability of groundwater for irrigation purpose is carried out by comparing the indices values with the Bureau of Indian Standards (BIS, 1986) as given in Table. 1. Indices exceeding the standards cause effects on soil health such as excess salt accumulation, increases pH and inhibits the movement of water into the root zone. Fig. 4 shows seasonal variation (post and pre-monsoon) of these irrigation indices. The percentage distribution as per two classification system (general and specific) is given the Table. 2. It was observed that, based on

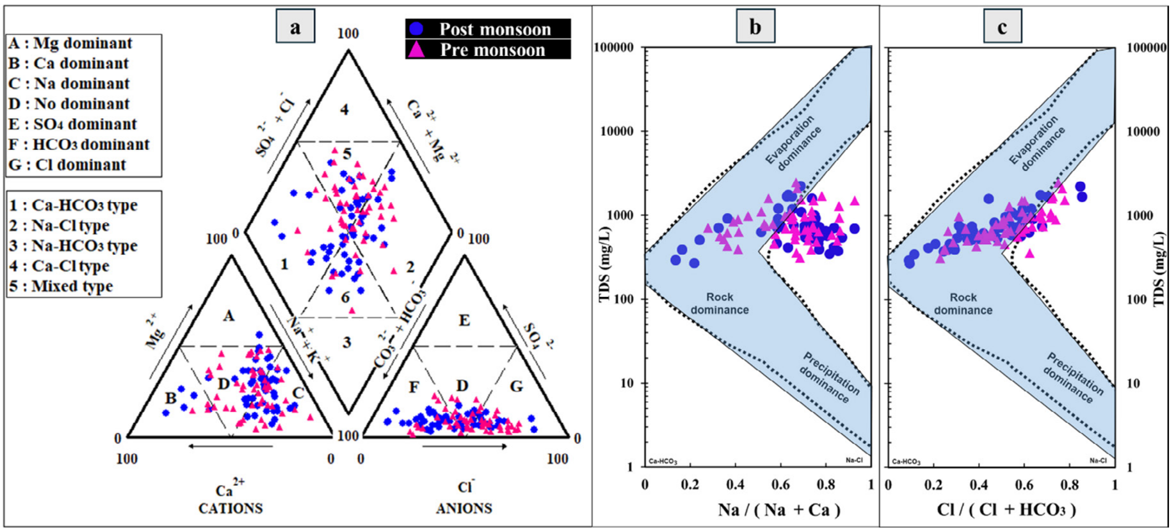

Fig. 3. (a) Piper diagram depicting the groundwater types and Gibbs diagrams based on ratios of (c) cation (b) anion to identify the dominance factors controlling groundwater chemistry in the study area

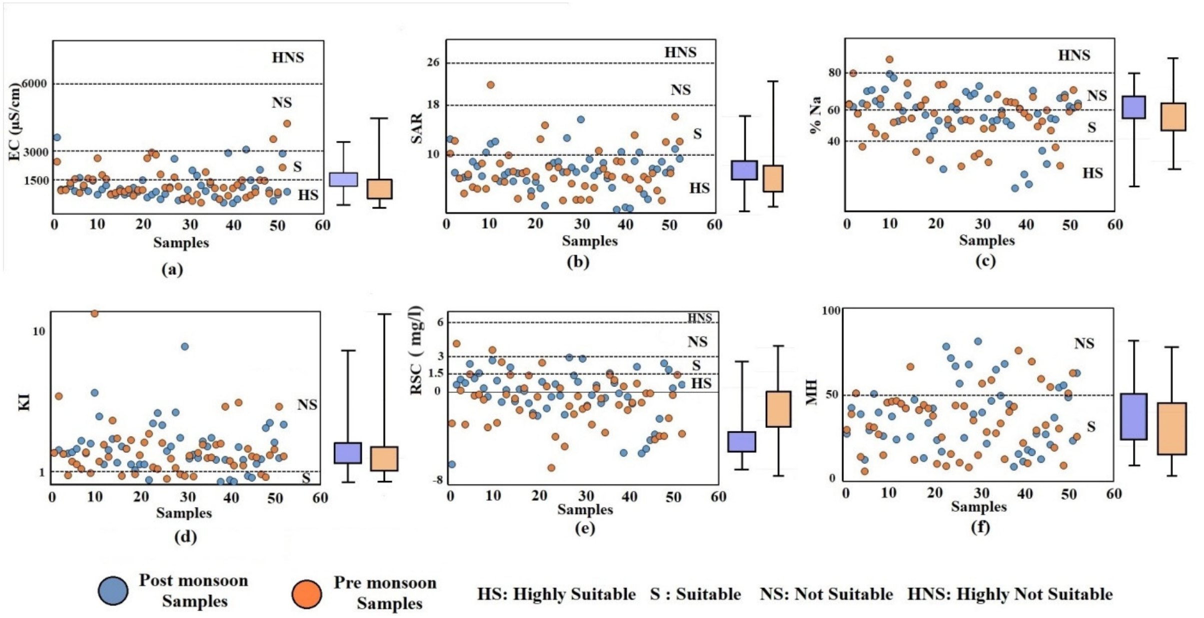

Fig. 4. Distribution of samples as per different irrigation indices (a) EC (b) SAR (c) %Na (d) KI (e) RSC (f) MH

Table 2. Percentage distribution of selected irrigation indices under general and specific classification

Classification	EC (µS/cm)		SAR		KI		% Na		RSC (mg/l)		MH	
Seasons	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*
Highly Suitable	76.9	63.5	82.7	80.8	-	-	11.5	17.3	80.8	90.4	-	-
Suitable	21.2	32.7	17.3	17.3	13.5	19.2	48.1	44.2	19.2	5.8	75	80.8
Not Suitable	1.9	3.8	0	1.9	86.5	80.8	40.4	36.6	0	3.8	25	19.2
Highly Not Suitable	0	0	0	0		-	0	1.9	0	0		-

the EC, SAR, KI, %Na, RSC, and MH indices values, 2%, 0%, 86.5%, 40%, 0% and 25% of groundwater samples during post monsoon and while 4%, 2%, 81%, 36.6%, 4%, and 19.2% groundwater samples during pre-monsoon are classified as the Not Suitable (N) for irrigation purpose. Each index shows different percentage of irrigation water quality classification, which provides unclear and indefinite decision about the suitability of water among indices. Further, Fig. 4 highlights that, water classification by indices is based on the crisp boundary values, thus the samples around the class boundaries are not completely clear of their class association

In order to address above uncertainties, a fuzzy based water quality index is computed by integrating the information derived from aforementioned parameters under thier uncertain boundary conditions.

Fuzzy Synthetic Evaluation for WQI

The combined effect of different hazards due to Salinity, Sodicity, Alkaline and Magnesium in irrigation water is evaluated using the Fuzzy Synthetic evaluation approach. From the Table. 1 it can be observed that the parameters SAR, %Na, RSC, EC consist of three limit values (therefore, four classes), whereas KI and MH consist of only one limit value (therefore, two classes). The classes are represented by membership functions between these standard limit values (λ). In order to construct membership functions, the values of λ_m^- and λ_m^+ are obtained using the Eq.5 and Eq.6 respectively for all the parameters. The inclusion of the degree of variation (δ) term, in the Eq.5 and 6, is a significant predictor of the outcome variable. It is important to note that, it allows the evaluator to choose the appropriate variation, based on the uncertainty information about the guideline limits, thus it increases the confidence about the evaluation results. When δ=0, in the evaluation, it denotes the crisp classification and therefore outcomes reflect those of the traditional water quality index. Further, the provision of 10- 20% degree of variation as per FAO guidelines (Ayers & Westcot, 1985) is divided into various extents (Ex. 0, 5, 10, and 20%) to obtain the μ values. The formation of membership function is given in the Fig. 5 for different parameters.

Further, the membership values are calculated using Eq.7-9, by adapting the same parameter guideline data as given in the Table. 1. Various fuzzy matrices were constructed, representing membership values of the parameters in columns (arranged in the order of EC, SAR, KI, %Na, RSC, and MH) corresponding to the classes in rows (arranged in the order of Highly Suitable, Suitable, Not Suitable and Highly Not Suitable) for each of the analyzed samples. The resultant matrices may include both fuzzy (μ values ranged between 0 and 1 under all the parameters) and non-fuzzy samples (μ values as either 1 or 0 under all the parameters). The details of fuzzy samples in all the indices are given in the Fig. 5.

Further, an integrated assessment of water quality by considering aforementioned parameters, a matrix is calculated by multiplying a weight matrix, as given in the Eq.12. The values of weightage were estimated by integrating subjective and objective knowledge of the sample data in this study. These findings demonstrate the significant differences between subjective and objective weights (Refer Supplemenatry Section 2), hence integrated weight is obtained by combining both weights using the weighted average method in order to obtain more accurate evaluation results, as given in the Table. 3. Based on this the priority of parameters is given

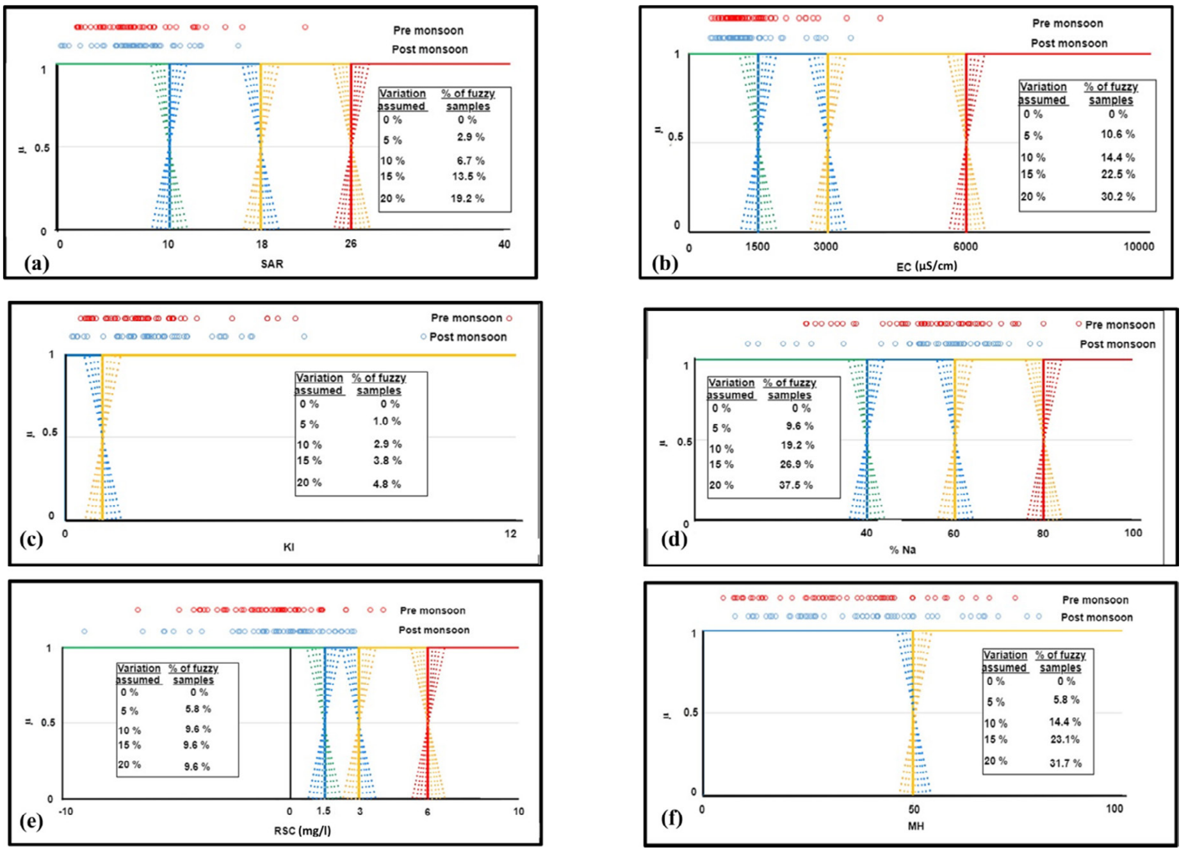

Fig. 5. Percentage of fuzziness in the data set, upon considering various boundary variation levels in the class boundaries of (a) EC (b) SAR (c) KI (d) % Na (e) RSC (f) MH

Table 3. Details of weightage values (ω)

Parameters	Relative weight (ω_{s1})		Relative weight (ω_{s2})		Subjectivity Weight (ω_{Sub})		Entropy (E)		Objectivity Weight (ω_{obj})		Integrated Weight (ω_{int})	
Seasons	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre
SAR	0.08	0.08	0.21	0.21	0.12	0.14	0.96	0.92	0.08	0.11	0.07	0.10
KI	0.38	0.42	0.07	0.07	0.2	0.24	0.93	0.90	0.12	0.14	0.18	0.21
%Na	0.23	0.17	0.21	0.21	0.36	0.29	0.97	0.96	0.05	0.06	0.13	0.11
RSC	0.08	0.08	0.21	0.21	0.12	0.14	0.75	0.66	0.50	0.47	0.44	0.43
MH	0.15	0.08	0.07	0.07	0.08	0.05	0.94	0.94	0.11	0.08	0.06	0.03
EC	0.08	0.08	0.21	0.21	0.12	0.14	0.92	0.91	0.14	0.13	0.12	0.12
Σ	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

in an order of RSC > KI >%Na > EC > SAR > MH. The weightage values are represented as a weight matrix (ω) as given Eq.13. Furthermore, in order to obtain final results in a Fuzzy evaluation matrix, the membership matrix is multiplied by weight matrix. An index is calculated by cumulating all the values in each row (i.e. each class). This index value gives the integrated information about the classification by various parameter. The final fuzzy index values of each sample are given in the Table. S2.

Fuzzy Based Classifications of Irrigation Water Quality

In the present study, two methods based on (i) Principle of maximum membership function (method-1) and (ii) ratio of fuzzy measure elements (method-2) were employed for the fuzzy based classification of water. Also, in both the methods, the traditional water quality index values were obtained by considering $\delta=0$ in the Eq. 5 and 6. A percentage distribution is shown in Table. S3 for the final classification of irrigation water based on a fuzzy comprehensive method of evaluation. According to the results of maximum membership degree principle (or method-1), the classification about the sample is obtained and the variations of samples distribution in each class are represented in the Fig. S3 (a) with respect to the assumed degree of variation (δ = 0%, 5%, 10%, 15% and 20%). It reveals that overall, 3.8 %, 3.8%, 2.9%, 3.8% and 2.9% samples as Not Suitable, corresponding to aforementioned δ values. Whereas, according to FCR results 0.98%, 0.98%, 1.96%, 1.96%, 1.96% are under Not Suitable classification, corresponding to different degree of variation (δ = 0%, 5%, 10%, 15% and 20%). The variation of sample distribution under each classification as per FCR approach (or method-2) at different δ values is given Fig. S3 (b).

However, the impact of considering the degree of variation or fuzziness is not clearly evident in the final classification using method-1. As depicted in Fig. S3(a), the classification variation, such as the percentage of samples classified under the High Suitable class, remains almost unchanged (83.6%) even when considering a δ value of 15%. This lack of discernible variation suggests that method-1 fails to capture the fuzziness in the evaluation process, as it only considers the maximum element value from the fuzzy matrix, neglecting the measured fuzzy information from other elements. Therefore, method-2, which utilizes all measured fuzzy information, is necessary. Fig. S3 (d) illustrates the distinct impact of uncertainties near the borderline on the final classification when using method-2. This variation may vary across different locations and is site-specific, demonstrating the influence of fuzziness near the border value on the final classification in this sample dataset. To investigate the impact of result derivation on final classification, Fig. S4 illustrates the distribution of non-fuzzy samples (μ = 1) and fuzzy samples ($0 < \mu < 1$). Initially, under traditional classification (δ=0), 42%, 32.8%, 25%, and 0.2% of samples fall into Highly Suitable, Suitable, Not Suitable, and Highly Not Suitable classes, respectively. As δ increases, the proportion of non-fuzzy samples decreases while fuzzy samples increase. Notably, the distribution of samples in the Suitable class is notably inconsistent due to their proximity to class borders in various indices (Fig. 4), signifying high uncertainty near these values, potentially impacting groundwater's final classification.

CONCLUSIONS

The study conducted in Koratagere and Madhugiri sub-districts of Tumkur district, Karnataka, aimed to assess groundwater suitability for irrigation using a fuzzy logic-based Water Quality Index (WQI). The analysis revealed that 54% of groundwater samples exhibited a mixed Na-Ca-HCO_3 water type, influenced primarily by rock-water interaction. Conventional indices classified post-monsoon samples with varying degrees of suitability, with notable disparities among individual indices. To address this, a fuzzy-based WQI incorporating boundary fuzziness was employed, considering different degrees of variation (δ=0%, 5%, 10%, 15%, and 20%). The final water classification was determined using two approaches: the Principle of Maximum Membership Function and Fuzzy Class Ratio. The latter effectively addressed boundary fuzziness in the dataset. Sample distributions near Suitable class boundaries revealed varying percentages of fuzzy samples at different δ values, indicating a positive correlation between sample data fuzziness and δ values. This underscores how fuzziness in sample distribution within specific indices can impact the final classification.

Overall, this study emphasizes a comprehensive approach to evaluating irrigation water quality, utilizing Fuzzy logic to handle uncertainty effectively, especially near boundary values. The developed methodology, by accounting for uncertainties within and among parameters, offers a more accurate assessment, aiding in sustainable agriculture practices. This method provides insights into water quality's impact on crop yield, guiding policymakers and irrigation managers towards sustainable groundwater management for future generations.

STATEMENTS & DECLARATION

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CONFLICT OF INTEREST

The authors declare that there are no financial or non-financial interests to disclose.

LIFE SCIENCE REPORTING

This study does not contain any studies with human participants or animals performed by any of the authors.

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